The Magnetic Connection Between the Convection Zone and Corona in the Quiet Sun

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ABSTRACT

To understand the dynamic, magnetic, and energetic connection between the convectively unstable layers below the visible surface of the Sun and the overlying solar corona, we have developed a new three-dimensional magnetohydrodynamic code capable of simultaneously evolving a model convection zone and corona within a single computational volume. As a first application of this numerical model, we present a series of simulations of the Quiet Sun in a domain that encompasses both the upper convection zone and low corona. We investigate whether the magnetic field generated by a convective surface dynamo can account for some of the observed properties of the Quiet Sun atmosphere. We find that (1) it is possible to heat a model corona to X-ray emitting temperatures with the magnetic fields generated from a convective dynamo and an empirically-based heating mechanism consistent with the observed relationship between X-ray emission and magnetic flux observed at the visible surface; (2) within the limitations of our numerical models of the Quiet Sun, resistive and viscous dissipation alone are insufficient to maintain a hot corona; (3) the Quiet Sun model chromosphere is a dynamic, non-force-free layer that exhibits a temperature reversal in the convective pattern in the relatively low-density layers above the photosphere; (4) the majority of the unsigned magnetic flux lies below the model photosphere in the convectively unstable portion of the domain; (5) horizontally-directed magnetic structures thread the low atmosphere, often connecting relatively distant concentrations of magnetic flux observed at the surface; and (6) low-resolution photospheric magnetograms can significantly underestimate the amount of unsigned magnetic flux threading the Quiet Sun photosphere.

Subject headings: Sun: corona — Sun: interior — Sun: magnetic fields — MHD

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1. Introduction

Understanding the physics of every important aspect of the solar magnetic field — from its generation and amplification in the turbulent, differentially rotating interior, to the ultimate release of magnetic energy in the solar atmosphere in the form of radiation, solar wind acceleration, flares, or coronal mass ejections — requires an understanding of the dynamic, magnetic, and energetic connection between the solar interior and the visible layers of the atmosphere extending from the photosphere out into the corona.

A quantitative understanding of this global connection is a formidable challenge. The convective interior is a high-density, optically-thick, turbulent regime, while the solar corona is a low-density, optically-thin, magnetically-dominated hot plasma. Magnetic fields entrained in convecting flows at and below the visible surface evolve relatively slowly compared to the coronal field, which can store energy over long periods of time, then undergo sudden, rapid, and dramatic topological changes as magnetic energy is released. Ultimately, this energy is introduced into the corona through the motion of magnetized plasma in the convective interior.

To date, most quantitative studies of the dynamics and energetics of evolving magnetic fields have focused separately on either the convective interior or the solar corona. This is because physical approximations can be made within each of these regimes that can greatly simplify the analysis. Unfortunately, most common simplifications of the physics do not apply to the highly-stratified, radiation-dominated magnetic transition between the upper convection zone and low-corona. As a result, the magnetic fields that thread this region are the least understood aspect of the globally-connected system.

There have been a number of attempts to simultaneously model the ideal MHD evolution of the upper convection zone and corona, mostly in the context of studies of emerging active regions (Fan 2001; Magara 2004; Manchester et al. 2004; Archontis et al. 2006; Magara 2006). While illuminating the physics of flux emergence, these investigations are highly idealized in the sense that the thermodynamic stratification of the atmosphere is imposed in an artificial manner. Since radiative processes, for example, are ignored in these models, the dynamics and energetics of plasma along coronal loops, and the effects of convective turbulence at and below the photospheric footpoints of those loops cannot be characterized in a physical way.

At the other end of the spectrum are more realistic three-dimensional MHD models of active region flux emergence through the turbulent surface layers (Cheung et al. 2007a), and of solar surface magnetoconvection (Bercik 2002; Stein & Nordlund 2002; Stein et al. 2003; Vögler et al. 2005; Schaffenberger et al. 2006; Stein & Nordlund 2006). In these calculations, the radiative source terms in the MHD energy equation are obtained via a direct
solution of the radiative transfer equation assuming LTE (local thermodynamic equilibrium). Simulations of this type admit to more direct comparisons with observations, but they are computationally expensive. This restricts the practical spatial scales over which they can be performed. Although Gudiksen & Nordlund (2002, 2005) have recently performed relatively large-scale, realistic simulations of the solar corona driven by a stochastic approximation of granulation, to date, realistic three-dimensional MHD models of surface magnetoconvection have yet to include the upper atmosphere (upper chromosphere, transition region and corona) within their computational domains.

Here, we take an intermediate approach. Our long-term goal is to bridge the gap between large-scale models of the solar magnetic field applicable to the deeper layers of the convective interior (e.g., Fan et al. 2003; Abbett et al. 2004; Browning et al. 2006), and models designed to describe the dynamic evolution of the outer corona (e.g., Roussev et al. 2004; Lionello et al. 2005; Lynch et al. 2006). Toward this end, we have developed a code capable of modeling the energetics of the turbulent upper convection zone and low-corona within a single computational domain. So that it remains feasible to extend the size of the domain to large spatial scales, our current approach is to eliminate the computationally expensive step of solving the transfer equation. Instead, we choose to approximate the treatment of optically thick surface cooling in a way that successfully reproduces the average stratification and solar-like convective turbulence of the more realistic simulations of Bercik (2002) and Stein et al. (2003).

In this paper, we present a first step toward bridging the gap between the solar interior and corona: a three-dimensional Cartesian MHD model of the Quiet Sun that extends from 2.5 Mm below the visible surface out into the low-corona. We focus this initial study on the magnetic connection between Quiet Sun magnetic fields threading the upper convection zone, and those magnetic fields filling the low-corona. We address several questions in particular: Can the magnetic field generated by a convective dynamo (Cattaneo 1999; Bercik et al. 2005) heat our model corona to X-ray emitting temperatures through physical processes such as resistive or viscous dissipation, or is an additional, empirically-based source of heating necessary (e.g., one consistent with the relationship of Pevtsov et al. 2003)? Do small-scale Quiet Sun magnetic fields exhibit a complex magnetic coupling between the photosphere and upper atmosphere as suggested by, e.g., Schrijver & Title (2003)? How do convectively generated fields interact with weaker, pre-existing coronal fields? Can the magnetic connectivity between the photosphere and corona be well described by static extrapolations?

The remaining portion of the paper is organized as follows. In Section 2 we describe our method of solution of the MHD system of equations with an emphasis on the numerical techniques necessary to evolve the combined system, and the energy source terms that ulti-
mately determine the thermodynamic stratification and energetics of the model atmosphere. In Section 3 we describe how we relaxed our initial convective state, and we describe how our simulations addressed the above questions. Finally, in Section 4 we discuss the limitations of our approach, and summarize our conclusions.

2. Method

The MHD system of equations is numerically solved on a three-dimensional Cartesian, block-adaptive mesh:

\[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0,\]

\[\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[ \rho \mathbf{u} \mathbf{u} + \left( p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\mathbf{BB}}{4\pi} - \mathbf{\Pi} \right] = \rho \mathbf{g},\]

\[\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{uB} - \mathbf{Bu}) = -\nabla \times (\eta \nabla \times \mathbf{B}),\]

\[\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{u}) = -p \nabla \cdot \mathbf{u} + \frac{\eta}{4\pi} |\nabla \times \mathbf{B}|^2 + \Phi + Q,\]

Here \(\rho\), \(\mathbf{u}\), \(e\), \(p\), \(\mathbf{B}\), and \(\mathbf{g}\) denote the gas density, velocity, internal energy per unit volume, gas pressure, magnetic field, and gravitational acceleration respectively (Gaussian units are assumed). The tensor \(\mathbf{\Pi}\) refers to the viscous stress tensor for a Newtonian fluid

\[\Pi_{ij} = 2\nu \left[ D_{ij} - \frac{1}{3}(\nabla \cdot \mathbf{u}) \delta_{ij} \right],\]

where \(D_{ij} = 1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)\) and \(\delta_{ij}\) refers to the Kronecker delta function. The dissipation function \(\Phi = \sum_{ij} \Pi_{ij} D_{ij}\) represents the rate of energy dissipation through viscous diffusion, and \(\nu\) and \(\eta\) (assumed constant) denote the coefficients of kinematic viscosity and magnetic diffusivity, respectively.

To close the system, we experiment with both an ideal-gas equation of state (with a ratio of specific heats \(\gamma = c_p/c_V\) of 5/3), and a somewhat more computationally expensive, but more physically realistic, non-ideal equation of state. The latter treatment is necessary in order to account for the effects of hydrogen and helium ionization on the internal energy, specific heat, and plasma \(\beta\) (the ratio of the gas to magnetic pressure) in the regions of the model atmosphere that extend from the hydrogen ionization zone below the photosphere out to the transition region. We use tabular data provided by the OPAL project (Rogers 2000), and choose photospheric values of the hydrogen and helium mass fractions (\(X = 0.75\) and \(Y = 0.23\)), and metallicity \(Z = 0.02\). We apply the OPAL equation of state throughout the computational domain from well below the photosphere out into the corona. We note that OPAL assumes LTE, an assumption that does not apply throughout much of the chromosphere. While this is a deficiency of our current approach, we are able to obtain
a more physical average thermodynamic stratification (determined by a comparison with
the non-LTE results of Abbett & Hawley 1999) using OPAL than by assuming an ideal,
fully-ionized hydrogen gas.

Although the OPAL data covers an impressive range of temperatures and densities,
there are times when a portion of the model atmosphere lies outside the range of the tables
(e.g., in low-density regions of the model corona that are relatively cool). To accommodate
the occasional gap in the tabular data, we extend the low density edges of the OPAL tables in
a way that smoothly transitions to an ideal treatment as necessary. We note that the regions
of the model atmosphere most acutely affected by the non-ideal treatment of the equation
of state lie near the visible surface, and that these layers have densities and internal energies
that are well within the range of the OPAL tables.

2.1. The Energy Source Terms

The energy source term \( Q \) contains all other sources of heating and cooling in the system
that are not otherwise explicitly represented in equation (4):

\[
Q = Q_r + Q_c + Q_B
\]  

This includes physical sources such as radiative cooling (denoted \( Q_r \), and described in Sec-
tion 2.1.1), and the divergence of the electron heat flux (the \( Q_c \) of Section 2.1.2), and any
empirically-based sources we choose to include in the system (e.g., the magnetic heating
\( Q_B \) of Section 2.1.3). We note that some of the physical source terms that we group into
equation (5) can be expressed in terms of a divergence of a physical flux, and in principle
could be included within the divergence term on the left-hand side of equation (4). However,
we treat these terms numerically as external sources to the energy equation (as described in
Section 2.2), and wish to remain consistent in our exposition. In the following sections, we
describe how we treat the physical and empirical source terms over the many physically dis-
tinct regions of the solar atmosphere and interior that are encompassed by our computational
domain.

2.1.1. Radiative Cooling

The radiative contribution to the energy equation, \( Q_r \), is expressed as a sum of three
terms, \( Q_1, Q_2, \) and \( Q_3 \), each applied to the appropriate sub-domain where the treatments
are valid. In the low-density upper transition region and corona, the radiative source term
is treated in the optically thin limit, and is expressed as
\[ Q_1 = -n_e n_h \Lambda(T). \] (6)

Here, \( n_e \) and \( n_h \) denote the electron and hydrogen number densities respectively, and can be expressed in the corona in terms of the gas density and mean molecular weight \( \mu = 1/(2X + 0.75Y + 0.5Z) = 0.59 \). For the runs that employ a non-ideal treatment of the equation of state, the temperature \( T \) is directly obtained from an inversion of the OPAL tables, given the plasma density and internal energy. In the ideal case, \( T \) is simply expressed as \( T = (\gamma - 1)e/(R^*\rho) \). Here, \( R^* = R/\mu \), where \( R \) denotes the universal gas constant, and \( p = (\gamma - 1)e \). The optically thin radiative cooling curve \( \Lambda(T) \) is generated from version 4.2 of the CHIANTI atomic database (Dere et al. 1997; Young et al. 2003) as described in Lundquist et al. (2007a). In addition, we include within \( \Lambda(T) \) the Peres et al. (1982) extension to the coronal cooling curve, applicable to the lower-temperature, higher-density plasma of the upper chromosphere.

In this study, we treat the optically-thick radiative cooling of the surface layers in an artificial way in order to eliminate the computationally expensive step of solving the radiative transfer equation in detail. While this limits our ability to compare our results directly to high-resolution observations of the Quiet Sun chromosphere (as was done in e.g., Carlsson et al. 2004; De Pontieu et al. 2006), it allows us to both maximize the spatial extent of the domain, and to run the simulations for the amount of time necessary to generate a Quiet Sun magnetic field directly from a weak seed field through the action of a convective dynamo.

We mimic surface losses in one of two ways. The first is to simply use the Peres et al. (1982) approximation to extend the range of applicability of the optically thin limit into the effectively thin low-chromosphere. The second is to apply a separate artificial source term throughout the low chromosphere of the form
\[ Q_2 = \tau^{-1}[e - e(\rho, T_0)]. \] (7)

Here, \( T_0 \) is a specified depth-dependent temperature profile, and \( \tau \) is a free parameter that sets the timescale over which this cooling is applied. The quantity \( e(\rho, T_0) \) is obtained directly from the tabular equation of state (or from the ideal gas equation of state if desired) given the local gas density, and a particular specification of \( T_0 \). We describe our choices for \( T_0, \tau \), and other relevant parameters below. Deeper in the model convection zone, we return to a more physical description of the radiative source term, and transition to the optically thick diffusion limit,
\[ Q_3 = \nabla \cdot (\kappa_R \nabla T), \] (8)

with a temperature and density-dependent Kramer’s opacity given by \( \kappa_R = \kappa_r T^{6.5} \rho^{-2} \) and \( \kappa_r = 1 \times 10^{-32} \) in cgs units (see Fan & Fisher 1996).
We thus express the total radiative contribution to the energy equation, \( Q_r \) of equation (5), explicitly as
\[
Q_r = \xi_1 Q_1 + \xi_2 Q_2 + \xi_3 Q_3.
\] (9)

The dimensionless envelope functions \( \xi_1, \xi_2, \) and \( \xi_3 \) restrict each term to an appropriate range of densities or depths in such a way as to avoid overly-sharp cutoffs. For the runs described in Section 3.1 (where we describe our calibration and relaxation process in detail), these functions take the form:
\[
\xi_1(\rho) = \frac{1 + \tanh[a_1(\rho - \rho_1)]}{2}, \quad \xi_2(z) = \frac{1 + \tanh[a_2l(z - z_{2l})]}{4}, \quad \text{and} \quad \xi_3(\rho) = \frac{1 - \tanh[a_3(\rho - \rho_3)]}{2}.
\]
The adjustable parameters \( \rho_1, z_{2l}, z_{2u}, \) and \( \rho_3 \) represent the cutoff values, while \( a_1, a_2l, a_2u, \) and \( a_3 \) control the steepness of the envelope function centered at those values. We choose these parameters (see Table 1 for a listing of the parameters used by the simulations introduced in Section 3.1), along with \( T_0 \) and \( \tau \) (e.g., \( T_0 \) is taken to be a constant value of 4500 K and \( \tau \equiv 10 \text{ s} \) for Run 2 of Section 3.1) so that our average stratification matches as closely as possible to that of the realistic simulations of Bercik (2002), where the radiative transfer of the surface layers was treated in a physical way. The radiative cooling of equation (9), coupled with a constant radiative flux or hydrostatic equilibrium thermodynamic lower boundary condition (on average), maintains the super-adiabatic stratification necessary to initiate and sustain turbulent convection.

2.1.2. Thermal Heat Flux in the Corona

Anisotropic thermal conduction from electrons plays an important role in the energy balance of the corona, and thus is included as the second source term in equation (5),
\[
Q_c = \xi \mathbf{B} \cdot \nabla \left( \kappa || \mathbf{B} \cdot \nabla T / B^2 \right).
\] (10)

Again, the dimensionless function \( \xi \) is a density-dependent envelope function of the form \( \xi(\rho) = \frac{1 - \tanh[a_4(\rho - \rho_4)]}{2} \) (the parameters \( a_4 \) and \( \rho_4 \) are listed in Table 1). Following Mikić et al. (2005), we define the temperature-dependent coefficient of thermal conductivity to be \( \kappa || = k_0 T^{5/2} \) when \( T \geq T_c \), \( (T_c \equiv 3 \times 10^5 \text{ K}) \), and \( \kappa || = k_0 T_c^{5/2} \) elsewhere in the transition region \( (k_0 = 1 \times 10^{-6} \text{ in cgs units}) \). In regions of the model corona where the field is weak (e.g., \( B_c < 0.001 \text{ G} \)), we transition to the standard hydrodynamic form of Spitzer (1962) conductivity, \( Q_c = \xi \nabla \cdot (\kappa || \nabla T) \).

So that the expected equilibrium between thermal conduction and radiative losses in coronal loops can be maintained, the corresponding optically thin loss function at temperatures below \( T_c \) is adjusted to include an additional multiplicative factor of \( (T/T_c)^{5/2} \) when \( T_{ch} < T < T_c \) \( (T_{ch} = 5 \times 10^4 \text{ is an adjustable parameter that represents the expected average} \).
temperature at the chromosphere-transition region boundary), and \((T_{ch}/T_c)^{5/2}\) for \(T \leq T_{ch}\). This avoids discontinuities in the cooling curve if applied at chromospheric temperatures. The Mikić et al. (2005) correction allows the transition region to be spread out in a physical way over a somewhat larger area, thereby eliminating the need to resolve the \(\sim 1\) km transition region scale height typical of models using the standard Spitzer conductivity (see e.g., Abbett & Hawley 1999).

### 2.1.3. An Empirically-based Coronal Heating Rate

The physical origin of the coronal heating mechanism is still a matter of debate. However, there is strong observational evidence that argues for a universal relationship between unsigned magnetic flux and the power dissipated through a coronal heating mechanism (Fisher et al. 1998; Pevtsov et al. 2003; Lundquist et al. 2007b; Warren & Winebarger 2006). This relationship gives us a straightforward prescription for calculating an empirically-based heating function that can be directly applied as a source term in the energy equation:

\[
\int Q_B dV = c\phi^\alpha. \tag{11}
\]

Here, the domain of integration is the sub-volume of the total domain lying above the visible surface. The right hand side of equation (11) is simply a restatement of the Pevtsov et al. (2003) power law relationship between X-ray luminosity and total unsigned magnetic flux \(\phi\) observed at the surface: \(L_x = c\phi^\alpha\). The values of \(c\) and \(\alpha\) are obtained empirically — we use those of Bercik et al. (2005): \(\alpha = 1.1488\) and \(c = 0.8940/\zeta\). Here, \(\zeta\) represents the fraction of the total energy dissipated that appears at soft X-ray wavelengths, \(\zeta = 1/100\) (see Lundquist 2006). If we choose a heating function of the form \(Q_B = a\psi\), where \(a\) is assumed constant, and \(\psi\) is a dimensionless spatially-varying weighting function, it immediately follows that \(a = c\phi^\alpha / \int \psi dV\). What remains is to specify \(\psi\) in such a way as to best reproduce the observational characteristics of coronal X-ray emission from the simulated data (as was done for force-free and potential field extrapolations by e.g., Schrijver et al. 2004; Lundquist et al. 2007b).

As it seems physically reasonable to assume that coronal heating is associated with the presence of magnetic fields, and that this heating is deposited in some manner along coronal loops (see Lundquist et al. 2007a), we choose a simple weighting function of the form \(\psi = B/\langle B \rangle\) where \(\langle B \rangle\) refers to the average magnetic field strength in the sub-volume lying above the visible surface. We can now express the coronal heating function explicitly as

\[
Q_B = \frac{c\phi^\alpha B}{\int B dV}, \tag{12}
\]
and include this as an optional contribution to the source term (equation [5]) included in equation (4). We note that our goal here is not to do a large parameter space exploration of the many different choices of weighting functions corresponding to plausible coronal heating mechanisms, rather it is simply to determine whether an empirically-based source term is capable of maintaining a hot model corona in the context of a 3D MHD model of the Quiet Sun.

2.2. RADMHD: A Numerical Solution to the Combined System

Obtaining a numerical solution to equations (1)-(4) over a computational domain that contains both a corona and a portion of the convective interior presents a number of significant challenges. The domain must span a 10-15 order of magnitude change in gas density, as well as the thermodynamic transition between the $\sim 1$ MK optically thin coronal plasma and the optically thick cooler layers of the lower atmosphere and interior. Furthermore, the corona is a low-density, magnetically-dominated plasma (we note that this may not be the case everywhere in the low corona, particularly in Quiet Sun atmosphere away from significant concentrations of magnetic flux), while the convection zone is a turbulent, high-$\beta$ plasma.

Below the surface, Quiet Sun magnetic fields evolve at the local convective turnover timescale and take the form of isolated, “spaghetti”-like structures with the strongest concentrations of magnetic flux located within intergranular lanes and strong penetrative downflows (see e.g., Stein & Nordlund 2002; Abbett et al. 2004; Bercik et al. 2005). In contrast, the low-$\beta$ field-filled corona often presents a simpler topology, but the plasma entrained within coronal loops evolves much more rapidly than the convective turnover time, and the coronal topology can suddenly and dramatically change in response to magnetic reconnection. In addition, the upper atmosphere tends to be shock-dominated (see e.g., Carlsson & Stein 1992; Abbett & Hawley 1999; Roussev et al. 2004), while typical sub-surface flow speeds remain well below the characteristic sound or Alfvén speed (allowing for the anelastic treatment of active region fields in the interior; see e.g., Fan et al. 1999).

To address much of the inherent spatial and temporal disparity of the system we developed the code RADMHD, which employs a unique combination of existing, well-studied algorithms to solve the MHD system semi-implicitly on a domain-decomposed computational grid. We note that although RADMHD has the capability to adaptively refine its Cartesian grid by interfacing with PARAMESH libraries (MacNeice et al. 2000), we did not use that capability during the course of this work. For the simulations presented in this paper, our choice of semi-implicit scheme is based on operator splitting, with a Crank-Nicholson tem-
poral discretization (see Press et al. 1986 for a discussion of these techniques). All other relevant numerical techniques are described below.

2.2.1. The Explicit Sub-step

The non-linear portion of the MHD system is treated explicitly in the absence of the energy source term $Q$, viscous stress, and magnetic resistivity (the gravitational source term in the momentum equation and the $-p\nabla \cdot \mathbf{u}$ contribution in the internal energy equation are also included in the explicit sub-step). We obtain a solution via the third-order accurate, semi-discrete central method of Kurganov & Levy (2000); Balbàs & Tadmor (2006), which we summarize below.

The discretized system of equations (1)-(4) can be expressed in conservative form as

\[
\frac{dq_{i,j,k}}{dt} = -\frac{H^x_{i+1/2,j,k}(t) - H^x_{i-1/2,j,k}(t)}{\Delta x} - \frac{H^y_{i,j+1/2,k}(t) - H^y_{i,j-1/2,k}(t)}{\Delta y} - \frac{H^z_{i,j,k+1/2}(t) - H^z_{i,j,k-1/2}(t)}{\Delta z} + S_{i,j,k}(t),
\]

where $q = (\rho, p, B, e)$ refers to the vector of conserved quantities averaged over each cell (at a discrete cell-centered location labeled $i, j, k$ in the $n_x n_y n_z$-sized computational block), and $H^x$, $H^y$, and $H^z$ denote the vector of corresponding numerical fluxes centered on the $x$, $y$, and $z$ faces of each cell (we note that the components of momentum $p_i = \rho u_i$ are evolved in our scheme, rather than the velocity itself). The vector $S$ represents source terms for each equation (if any) that are included in the explicit sub-step. The Cartesian grid is assumed uniform within a given block of the domain-decomposed domain, and the cell sizes $\Delta x$, $\Delta y$, and $\Delta z$ are equal and constant within each sub-domain.

The face-centered numerical fluxes are given by:

\[
H^x_{i+1/2,j,k}(t) = \frac{F^x(q^+_{i+1/2,j,k}(t)) + F^x(q^-_{i+1/2,j,k}(t))}{2} - \frac{\alpha^x_{i+1/2,j,k}(t)}{2} \left[ q^+_{i+1/2,j,k}(t) - q^-_{i+1/2,j,k}(t) \right]
\]

\[
H^y_{i,j+1/2,k}(t) = \frac{F^y(q^+_{i,j+1/2,k}(t)) + F^y(q^-_{i,j+1/2,k}(t))}{2} - \frac{\alpha^y_{i,j+1/2,k}(t)}{2} \left[ q^+_{i,j+1/2,k}(t) - q^-_{i,j+1/2,k}(t) \right]
\]
\[
H_{i,j,k+1/2}(t) = \frac{F^z(q_{i,j,k+1/2}^+(t)) + F^z(q_{i,j,k+1/2}^-(t))}{2} - \frac{a_{i,j,k+1/2}(t)}{2} \left[ q_{i,j,k+1/2}^+(t) - q_{i,j,k+1/2}^-(t) \right],
\]

where the elements of \(F^x, F^y,\) and \(F^z\) are the physical fluxes corresponding to each element of the state vector \(q\). The quantities \(a^x, a^y,\) and \(a^z\) refer to the maximum propagation speed of discontinuities locally at each cell face in the \(x, y\) and \(z\)-directions respectively \([\text{i.e., the unsigned maximum of the eigenvalues of the Jacobian matrices of } F^x(q), F^y(q),\) and \(F^z(q)\)]\). In practice, these are set by the maximum of the limiting values of the fast magnetosonic wave speed on either side of every cell face (as described below). In the semi-discrete formalism of Kurganov & Levy (2000), the limiting values of the state vector are obtained via piecewise polynomial reconstructions centered on either side of a cell face; for example, in the \(x\)-direction

\[
q_{i+1/2,j,k}^+(t) = P_{i+1,j,k}(x_{i+1/2}, y_j, z_k, t)
\]

\[
q_{i+1/2,j,k}^-(t) = P_{i,j,k}(x_{i+1/2}, y_j, z_k, t).
\]

The semi-discrete formalism is independent of the choice of a particular reconstruction technique; here, we choose the third-order accurate, central weighted essentially non-oscillatory (CWENO) reconstruction of Levy et al. (2000) to determine the form of the interpolating polynomials \(P_{i,j,k}(x, y, z, t)\) along each direction. We summarize this technique below.

### 2.2.2. CWENO Interpolation

For each element of the state vector, we wish to define an interpolating polynomial

\[
P(x, t) \equiv P_{i,j,k}(x, y, z, t)
\]

as a linear combination of a polynomial accurate to third order (also centered at \(i, j, k\)), \(Q_C(x, t)\), and two second order accurate polynomials \(Q_L(x, t)\) and \(Q_R(x, t)\),

\[
P(x, t) = w_L Q_L(x, t) + w_C Q_C(x, t) + w_R Q_R(x, t).
\]

The CWENO coefficients \(w_L, w_C,\) and \(w_R\) sum to unity, and are designed to favor the high-order interpolating polynomial in most cases, and the appropriate low-order polynomial as necessary to minimize any spurious oscillations that may develop near shocks. We will discuss the explicit form of these weights after deriving the interpolating polynomials below.

Consider first a multi-dimensional Taylor expansion about \(x = a\) of the function \(q(x, t)\) — a continuous representation of a single component of the state vector whose values at a time \(t = t_n\) are known only at discrete locations within the computational domain (denoted
henceforth as $q_{i,j,k}^n$). To arbitrary accuracy, we have

$$q(x, t_n) = \sum_{m=0}^{M} \left\{ \frac{1}{m!} [(x - a) \cdot \nabla']^m q(x', t_n) \right\}_{x' = a}, \quad (19)$$

where the operator $\nabla'$ denotes the gradient operator with respect to $x'$. Enforcing the conservation of a known cell average over its control volume $V$ requires that

$$q_{i,j,k}^n = \frac{1}{\Delta V} \int_V q(x, t_n) \, dV. \quad (20)$$

Truncating the Taylor series of equation (19) to achieve a desired accuracy, and performing the above integration over a sufficient number of surrounding points results in a system of equations that determines the value of the function and its derivatives at $x = a$, and yields an approximation for $q(x, t_n)$ in terms of known cell averages evaluated at nearby points.

If we choose $M = 2$, assume $x = a$ corresponds to the value of $x$ at the mesh point $i, j, k$, and neglect the cross-terms of equation (19) (a form of the expansion valid only along coordinate axes centered at $i, j, k$), an evaluation of equation (20) for $q_{i,j,k}^n, q_{i+1,j,k}^n, q_{i-1,j,k}^n, q_{i,j+1,k}^n, q_{i,j-1,k}^n, q_{i,j,k+1}^n, and q_{i,j,k-1}$ yields a simple, computationally efficient compact stencil accurate to third order, which in the $x$-direction takes the form

$$q(x, y_j, z_k, t_n) = A + B(x - x_i) + C(x - x_i)^2 \quad (21)$$

with the coefficient $A \equiv q(x_i, y_j, z_k, t_n)$ given by

$$A = q_{i,j,k}^n + \frac{1}{24} (q_{i+1,j,k}^n - 2q_{i,j,k}^n + q_{i-1,j,k}^n)
+ \frac{1}{24} (q_{i,j+1,k}^n - 2q_{i,j,k}^n + q_{i,j-1,k}^n)
+ \frac{1}{24} (q_{i,j,k+1}^n - 2q_{i,j,k}^n + q_{i,j,k-1}^n), \quad (22)$$

and the derivatives $B \equiv q_x(x_i, y_j, z_k, t_n)$ and $C \equiv q_{xx}(x_i, y_j, z_k, t_n)$ given by the familiar differencing formulae

$$B = \frac{q_{i+1,j,k}^n - q_{i-1,j,k}^n}{2\Delta x}, \quad (23)$$

and

$$C = \frac{q_{i+1,j,k}^n - 2q_{i,j,k}^n + q_{i-1,j,k}^n}{(\Delta x)^2}. \quad (24)$$

Similar expressions hold along the $y$ and $z$ coordinate axes centered at $i, j, k$. Henceforth, for brevity, we explicitly show the differencing formulae valid only along the $x$-axis; and
unless otherwise stated, it is implicitly understood that similar expressions hold in the other Cartesian directions.

We now repeat the integration of equation (20) for a Taylor series expansion truncated at \( M = 1 \) for the points \( q^n_{i,j,k}, q^n_{i-1,j,k}, q^n_{i,j-1,k}, \) and \( q^n_{i,j,k-1} \) and readily obtain the second order accurate approximation

\[
Q_L(x, y_j, z_k, t_n) = q^n_{i,j,k} + \frac{1}{\Delta x} (q^n_{i,j,k} - q^n_{i-1,j,k}) (x - x_i). \tag{25}
\]

For the points \( q^n_{i,j,k}, q^n_{i+1,j,k}, q^n_{i,j+1,k}, \) and \( q^n_{i,j,k+1} \) we obtain the other one-sided stencil

\[
Q_R(x, y_j, z_k, t_n) = q^n_{i,j,k} + \frac{1}{\Delta x} (q^n_{i+1,j,k} - q^n_{i,j,k}) (x - x_i). \tag{26}
\]

If we require that the stencil \( q(x, y_j, z_k, t_n) \) of equation (21) be expressed as a linear combination of \( Q_L, Q_C, \) and \( Q_R, \)

\[
q(x, y_j, z_k, t_n) = c_L Q_L(x, y_j, z_k, t_n) + c_C Q_C(x, y_j, z_k, t_n) + c_R Q_R(x, y, z_k, t_n), \tag{27}
\]

we can determine the form of the centered polynomial \( Q_C. \) With a symmetric choice of \( c_L = c_R = 1/4 \) and \( c_C = 1/2, \) we obtain

\[
Q_C(x, y_j, z_k, t_n) = q^n_{i,j,k} - \frac{1}{12} (q^n_{i+1,j,k} - 2q^n_{i,j,k} + q^n_{i-1,j,k})
- \frac{1}{12} (q^n_{i,j+1,k} - 2q^n_{i,j,k} + q^n_{i,j-1,k})
- \frac{1}{12} (q^n_{i,j,k+1} - 2q^n_{i,j,k} + q^n_{i,j,k-1})
+ \frac{1}{2\Delta x} (q^n_{i+1,j,k} - q^n_{i-1,j,k}) (x - x_i)
+ \frac{1}{(\Delta x)^2} (q^n_{i+1,j,k} - 2q^n_{i,j,k} + q^n_{i-1,j,k}) (x - x_i)^2 \tag{28}
\]

along the \( x \)-coordinate axis centered at \( i, j, k. \) We note that the above expressions are equivalent to those obtained using the “dimension-by-dimension” approach of Kurganov & Levy (2000).

What remains is to specify the non-linear CWENO weights of equation (18). For a given component of the state vector, we use the non-linear weighting functions introduced by Jiang & Shu (1996) and discussed by Levy et al. (1999) (and references therein),

\[
w_I = \frac{\xi_I}{\sum_J \xi_J}. \tag{29}
\]
Here, the indices $I$ and $J$ refer to the left, center, and right subscripts $L$, $C$, and $R$, with $\xi_I$ given by

$$\xi_I = \frac{c_I}{(\epsilon + S_I)^p}. \quad (30)$$

The coefficients $c_I$ are, as before, chosen to be $c_L = c_R = 1/4$ and $c_C = 1/2$, the constant $\epsilon$ is assumed negligibly small to prevent division by zero in certain cases, and the exponent $p$ (an adjustable parameter in the code) is set to 2 for all the runs presented in this paper.

We follow Jiang & Shu (1996); Zhang & Shu (2003) and define the measure of “smoothness”, $S_I$, for a given component of the state vector within the control volume $V_{i,j,k}$ as

$$S_I = \sum_{1 \leq |\alpha| \leq M} \frac{1}{\Delta V} \int_{V_{i,j,k}} [\Delta x^\alpha D^\alpha Q_I(x,t)]^2 dV. \quad (31)$$

Here, multi-index notation is used: $\alpha$ represents the three component multi-index $\alpha = (\alpha_1, \alpha_2, \alpha_3)$, $|\alpha| = \alpha_1 + \alpha_2 + \alpha_3$ denotes the sum of its components, $\Delta x^\alpha \equiv \Delta x^{\alpha_1} \Delta y^{\alpha_2} \Delta z^{\alpha_3}$, and $D^\alpha \equiv \partial^{\alpha_1} / (\partial x^{\alpha_1} \partial y^{\alpha_2} \partial z^{\alpha_3})$. The upper bound $M$ refers to the order of the polynomial $Q_I$.

If we construct smoothness indicators separately for each Cartesian direction, equation (31) along the $x$-axis reduces to the smoothness indicators of Levy et al. (1999):

$$S_I = \int_{x_{i-1/2}}^{x_{i+1/2}} \left[ \Delta x \left( \frac{d}{dx} Q_I(x, y_j, z_k, t_n) \right)^2 + (\Delta x)^3 \left( \frac{d^2}{dx^2} Q_I(x, y_j, z_k, t_n) \right)^2 \right] dx. \quad (32)$$

A straightforward calculation involving equations (25), (26), (28), and (31) or (32) yields an explicit form of the smoothness indicators along the $x$-axis centered at $i, j, k$:

$$S_L = (q^n_{i,j,k} - q^n_{i-1,j,k})^2$$

$$S_R = (q^n_{i+1,j,k} - q^n_{i,j,k})^2$$

$$S_C = \frac{1}{4} (q^n_{i+1,j,k} - q^n_{i-1,j,k})^2 + \frac{13}{3} \left( q^n_{i+1,j,k} - 2q^n_{i,j,k} + q^n_{i-1,j,k} \right)^2 \quad (33)$$


The question now becomes how best to define smoothness indicators for the coupled MHD system. We find that the optimal strategy largely depends on the particular problem at hand, and to some extent on how the conserved quantities are normalized or scaled relative to one another (if at all). No one technique works optimally well for all problems. There are a number of approaches, ranging from using different weights for each conserved quantity, to implementing global smoothness parameters (those that depend on a global norm over all elements of the state vector), to simply basing the smoothness parameter for the system on one, physically relevant component of the state vector. We find that for the
simulations presented in this paper (where there is a steep average density profile in the dynamic transition layers between the photosphere and corona), the latter approach works best, and we define smoothness parameters for the system based on the gas density alone.

With the specification of the weights, the interpolating polynomial for each component of the state vector \( P(x, y, z, t) \equiv P_{i,j,k}(x, y, z, t) \) of equation (18) is fully specified at \( t = t_n \) in terms of known quantities \( q_{i,j,k}^n \). We can now interpolate a given cell-centered component of the state vector to any given face center of its control volume. Thus, the required limiting values of the state vector on either side of a cell face (equation [17]) are defined, the numerical fluxes of equations (14)-(16) can be calculated, and, given a temporal discretization, the vector of conserved quantities can be updated.

The advantage of the explicit semi-discrete CWENO treatment lies primarily in its simplicity — the approximate Riemann problem need not be solved, nor is a characteristic decomposition required. The CWENO formalism is designed to conserve cell averages, and to enforce the least oscillatory behavior possible around discontinuities (any small oscillations that arise from interpolation error decay away) while maintaining (in this case) spatial accuracy to third order in smooth regions. The cell-centered, compact stencil greatly simplifies the implementation of boundary conditions, and is easily incorporated into a domain decomposition framework.

A drawback of this, and other non-staggered central schemes is that there is no guarantee that the magnetic field will remain divergence-free to round-off error (as is the case with e.g., the constrained transport scheme of Evans & Hawley 1988; Stone & Norman 1992). To eliminate the buildup of numerical error, we add an artificial source term to the MHD induction equation proportional to \( \nabla (\nabla \cdot B) \) and incorporate this term into our implicit sub-step. This approach corresponds to solving the constrained MHD system of equations using an implicit parabolic correction (see Dedner et al. 2003), and acts to dissipate any local divergence errors out of the domain given an appropriate transmitting external boundary condition. For the simulations presented in this paper, we find that the maximum relative divergence error \( \nabla \cdot B / |B| \) per pixel rarely exceeds \( 10^{-6} \). We have evaluated the performance of our explicit shock capturing scheme using a number of standard one, two, and three-dimensional advection and shock tests (see, e.g., Brio & Wu 1988; Orszag & Tang 1998; Toth 2000 and references therein), and our results compare favorably with similar tests of other high-order schemes (e.g., Toth 2000; Balbàs & Tadmor 2006).
2.2.3. The Implicit Sub-step

The effects of resistive and viscous dissipation, the contributions of the remaining energy source terms, and the divergence correction are treated implicitly via a “Jacobian-free” Newton-Krylov solver. A detailed description of Jacobian-free methods can be found in Knoll & Keyes (2003) (and references therein). Briefly, the solver is based upon a multi-dimensional Taylor expansion of \( F(q) \) (where \( F(q) = 0 \) represents the system of evolution equations for the implicit sub-step) about the state vector \( q \), whose components are the conserved quantities of the MHD system:

\[
F_i(q + \delta q) = F_i(q) + \sum_{j=1}^{N} J_{ij}(q) \delta q_j + O(\delta q^2)
\]  

(34)

The components of the Jacobian matrix are defined as \( J_{ij}(q) = \frac{\partial F_i(q)}{\partial q_j} \), and the indices \( i \) and \( j \) span the linear dimension of the system \( N = n_{eq} n_x n_y n_z \) (\( n_{eq} \) denotes the number of components in the state vector \( q \), and \( n_x \), \( n_y \), and \( n_z \) denote the number of cells in each Cartesian direction). If we set \( F(q + \delta q) \) to zero in equation (34), and neglect terms of order \( (\delta q)^2 \) and higher, we obtain the standard Newton method, an iterative procedure to obtain the solution to a non-linear system given an initial guess for the state vector:

\[
F(q) = -J(q)\delta q
\]

(35)

(here, the equation is expressed in matrix notation). Once \( \delta q \) is determined, we update the current state vector \( q \) via \( q^{\text{new}} = q + \delta q \). The procedure is repeated with the updated state vector serving as the new initial state until such time as the normalized correction \( ||\delta q/q|| \) reaches an acceptable error tolerance \( \epsilon \) (taken to be \( 10^{-6} \)).

For large three-dimensional systems, the standard approach of specifying the components of the Jacobian matrix and directly inverting equation (35) to solve for the correction vector \( \delta q \) can be computationally prohibitive. For example, for non-sparse systems, the amount of memory required to store the components of the \( N \times N \) Jacobian matrix can be excessively large (recall \( N = n_{eq} n_x n_y n_z \). To address this limitation (and others), it is useful to employ Krylov-based techniques to solve the linear system of equation (35).

Beginning with an initial residual \( r_0 = F(q) + J(q)\delta q_0 \), and an initial guess for \( \delta q_0 \) (assumed zero, since the normalized correction vector of the previous Newton iteration is presumably small), a Krylov subspace method will generate an approximation to the correction vector (for a given Newton iteration) of the form

\[
\delta q_j = \delta q_0 + \sum_{i=0}^{j-1} \alpha_i \left[ J(q) \right]^i r_0
\]  

(36)
for $j > 0$. If the coefficients $\alpha_i$ are chosen to minimize the residual norm for a given $j$, the approximation becomes increasingly accurate for larger $j$. The advantage of Krylov methods is evident in equation (36): each successive approximation after $j = 1$ requires only the specification and storage of a matrix-vector product involving the Jacobian, rather than the Jacobian itself (assuming each of the preceding, less accurate approximations are stored in memory).

The Krylov method employed here is the widely-used, generalized minimum residual (GMRES) method originally introduced by Saad & Schultz (1986), and described in detail by, e.g., Ipsen & Meyer (1998); Knoll & Keyes (2003). The $j$-th GMRES iteration minimizes the residual norm $||F(q) + J(q)\delta q_j||$ by using Arnoldi’s method (a modification of the Gram-Schmidt orthonormalization procedure tailored to Krylov sub-spaces) to express $\delta q_j$ in terms of a more convenient, orthonormal basis. This allows for the minimization above to be expressed as a standard least-squares problem, the solution to which can be obtained at relatively low computational cost. A drawback of the GMRES method is that the basis vectors from previous GMRES iterations must be stored (a disadvantage shared with other, similar Krylov subspace techniques). In practice, we set a maximum size of the Krylov subspace $j_{\text{max}} = m$ ($m$ is an adjustable parameter usually set to 10). If the final estimate of the residual is above a prescribed error tolerance, the Krylov sub-step is restarted with a non-zero $\delta q_0$, and is allowed to proceed for another $m$ iterations. Of course, only a finite number of restarts are allowed.

What remains is to approximate a matrix-vector product of the form $J(q)v$, where the vector $v$ is a known quantity (e.g., obtained from the basis vectors of a previous GMRES iteration). We follow the method outlined in Knoll & Keyes (2003) and construct a first-order Taylor expansion of $J(q)v$, which leads to the approximation

$$J(q)v \sim \frac{F(q + \epsilon v) - F(q)}{\epsilon},$$

(37)

where $\epsilon$ is a small quantity. To aid with scaling, and to mitigate roundoff error, we choose $\epsilon = \delta \sum_{n=1}^{N} (q_n + 1)/||v||$, with a choice of $\delta = 10^{-9}$. It is important to note that the order of accuracy of the correction vector does not affect the order of accuracy of the solution; it only affects the rate of convergence of Newton’s method.

A few additional comments are in order. Although the GMRES method is guaranteed to converge, its convergence rate is not always optimal. This is especially true for stiff systems of equations where the chosen timestep greatly exceeds the explicit CFL limit. To address this problem, it is necessary to either implement a pre-conditioner that forces the matrix system of equations (or approximations thereof) into a more diagonally-dominant state (see e.g., Chacon & Knoll 2003), or employ other techniques capable of efficiently treating the
system (see, e.g., Tokman & Bellan 2002). In this study, we are content to restrict ourselves to the CFL limit imposed by fast magnetosonic waves in the corona, and find that the number of Krylov iterations necessary to generate an acceptable approximation for $\delta q^k$ in our implicit sub-step rarely exceeds 10 (even though our timestep restriction exceeds, e.g., the characteristic conductive and radiative timescales). In principle, we could use the Newton-Krylov technique alone to implicitly solve the entire system, with our explicit techniques as physics-based pre-conditioners for the system. Although we do not pursue that option in this paper, it is something that we will explore in the near future.

3. Results

As an initial application of RADMHD, we now present a series of simulations of the Quiet Sun magnetic field generated by the action of a convective dynamo. We describe the magnetic connectivity between the layers below the surface and the model corona, and investigate whether an empirically-based coronal heating mechanism is required to maintain a hot model corona. What follows is a brief overview of the relaxation procedure of our combined convection zone-to-corona models, a description of the thermodynamic and magnetic structure of these simulations, a description of the distribution of magnetic flux at the visible surface, and a brief discussion of the effect of resistive dissipation on the energetics of the model corona.

3.1. Building the Model Convection Zone

Our first task is to dynamically and energetically relax the convectively unstable layers of the computational domain in the absence of a magnetic field. There are a number of ways to achieve this objective; what follows is a brief recipe that we find accelerates this often tedious, computationally expensive process. So that the physical convective relaxation time is not excessively long, we choose to include within our domain only the near-surface portion of the convective interior extending 2.5 Mm below the visible surface.

We begin by constructing a long, narrow, rectangular domain in the vertical direction of resolution $2 \times 2 \times 64$, and vertical extent of 7.5 Mm. We assign periodic boundary conditions in both horizontal directions, and a closed, anti-symmetric boundary at the top of the box. The lower boundary is also closed, but two additional requirements are imposed: the gradient of the gas density between the first and second active layers is maintained across the boundary, and the internal energy per unit volume is set to maintain an average temperature gradient.
across the boundary consistent with the simulations of Bercik (2002) \( \frac{dT}{dz} \sim -350 \text{ K/km}, \)
for \( z \) increasing with height). We then activate the radiative source terms in the energy equation, artificially damp strong vertical oscillations, set the coefficients of viscosity and resistivity to zero, and relax a vertically-stratified, horizontally-invariant background state.

The location of the visible surface, and the run of superadiabaticity with depth below the surface is sensitive to the choice of parameters in our idealized treatment of optically thick radiative cooling (as described in Section 2). To ensure that our artificial cooling behaves as realistically as possible, we calibrate the free parameters so that: 1. the resulting sub-surface stratification matches as closely as possible to the average stratification found in the simulations of Bercik (2002), where the LTE transfer equation is solved in detail; and 2. the resulting chromospheric densities and pressures are as consistent as possible with the quiescent model atmosphere of Abbett & Hawley (1999), where the non-LTE radiative hydrodynamic equations (for a plane-parallel atmosphere) were solved in detail. This ensures that solar-like convective turbulence can be initiated and maintained throughout the sub-surface portion of the computational domain, and that the low atmosphere retains, on average, a solar-like stratification.

In the calibration runs where an ideal gas equation of state is used, we find that it is possible, using strong surface cooling, to obtain temperature and density profiles and a run of super-adiabaticity with height from the upper convection zone into the low atmosphere that compare favorably with the Bercik (2002) results. However, it is not possible to simultaneously match the pressure stratification near the surface. This is not surprising, since the reference simulations include a more realistic treatment of the equation of state. Unrealistic values of pressure near the surface will affect the magnetic connection between sub-surface and coronal fields most acutely if the excessively high gas pressure causes appreciable changes in the plasma-\( \beta \) around strong concentrations of magnetic flux. The fact that we could not rule out this possibility motivated us to incorporate into the code the option to use the OPAL equation of state (as described in Section 2). Of course, when we relax an initial stratification using the non-ideal equation of state, the pressure stratification at and above the surface relaxes to values that compare favorably to the reference models.

Next, we extend the horizontally-invariant atmospheres to a \( 30 \times 30 \times 7.5 \text{ Mm}^3 \), relatively low-resolution \( 256 \times 256 \times 64 \) cube (distributed over 16 processors), greatly reduce the artificial damping, impose a zero-gradient upper boundary condition on all components of the state vector (note that at this point, the model corona is a cold, nearly evacuated region), and break the one-dimensional symmetry by adding random internal energy perturbations throughout the portion of the domain representing the convective interior. Since the model convection zone is, by construction, super-adiabatically stratified, convection begins immediately. Once
initiated, the domain is allowed to dynamically and energetically relax to a steady state. During this time, the vertical damping is gradually eliminated. For the runs presented in this paper, we continue to use a standard anti-symmetric lower boundary condition on the magnetic field. We note that while our vertical boundary conditions on the magnetic field are easy to implement, they do have physical consequences (as discussed in Section 4). We are currently working on improving our treatment of the upper and lower boundaries, and plan to report on our progress in the near future.

We then introduce an energetically unimportant magnetic seed field into the model convection zone (of 0.01 G in the \( x \)-direction), and activate all remaining energetic source terms, with the exception of thermal conduction in the corona. Since we wish to investigate the effects of viscous and resistive heating separately from the effects of empirically-based coronal heating, we choose to set the coefficients of resistivity and viscosity, \( \mu \) and \( \nu \), to zero when the empirical heating is active. The convective dynamo (Cattaneo 1999; Bercik et al. 2005) begins to operate, and magnetic energy within the domain increases with time. Although only a small amount of magnetic flux enters the model corona, it begins to heat up as the dynamo is allowed to saturate. Finally, we activate the thermal conduction source term, and allow the corona to equilibrate.

### 3.2. The Thermodynamic Structure of the Quiet Sun Model Atmosphere

Although we experimented with a number of relaxation procedures, choices of boundary conditions, free parameters for the optically thick cooling terms, and weighting functions for the empirical coronal heating source term, we focus here on two simulations that best maintain a combined solar-like convection zone and corona within a single computational domain. They were generated in the manner described above, and differ in the following way: The first run (hereafter referred to as Run 1) employs an ideal equation of state and approximates all surface cooling by extending the range of applicability of the Peres et al. (1982) extension of the coronal cooling curve to the low-chromosphere as described in Section 2.1.1.

The second run (Run 2) uses the OPAL equation of state, and approximates surface cooling with the additional radiative source source term (the second term in equation [9]) described in Section 2.1.1. The envelope function parameters for each run are listed in Table 1. In addition, since Run 2 was initially intended as a simple check to ensure that the ideal equation of state did not introduce any unphysical changes in the magnetic structure of the combined system near the surface, we expedited the dynamo process during a time of slow growth by increasing the magnetic field strength by an order of magnitude. We then let
the process continue until the average strength of the magnetic field at the surface in Run 2 was comparable to that of Run 1, which achieved a ratio of total magnetic to total kinetic energy in the convective portion of the domain of 0.06 purely as a result of the action of the convective dynamo.

Of course, both runs are highly idealized in the sense that the radiative transfer is not treated in detail. Normally, in more realistic simulations (see e.g., Abbett & Hawley 1999), the location of a physical chromospheric temperature minimum is easily identified, and the visible surface can be defined, for example, in terms of the average height where the plasma becomes optically thin to continuum radiation. The different regions of the solar atmosphere can thus be defined in a relatively straightforward way — the photosphere extends from the visible surface to the temperature minimum, the chromosphere extends from the temperature minimum out to the high-temperature low-density transition region, and so on. In our case, however, we are unable to define a temperature minimum or a visible surface in a physically meaningful way.

We must therefore characterize our model atmosphere in a somewhat arbitrary (yet self-consistent) manner. We set the location of the “visible surface” at the height where the horizontally-averaged gas pressure reaches a typical photospheric value, $p_s$. We define this characteristic surface pressure as that which is necessary to maintain a plasma-$\beta$ of unity in the presence of a kilogauss-strength concentration of magnetic field (e.g., $p_s = 3.5 \times 10^{-5}$ dyn cm$^{-2}$ for Run 2). Similarly, we set the location of the transition region boundary at the height where the horizontally-averaged gas temperature reaches the expected value $T_{ch}$, as defined in Section 2.1.2.

We define the model “chromosphere” to be the dynamic, but convectively stable region above the surface overshoot layer and below the transition region boundary, and we refer to the remaining layers below the chromosphere but above the visible surface as the model photosphere. We stress once again that we do not treat the physics of the chromosphere in a realistic fashion. Nonetheless, we find that in the context of our current model, it is useful to define a visible surface and chromosphere in terms of the magnetic properties of the plasma, since it allows us to clearly and concisely describe the complex magnetic and energetic transition between the sub-surface layers and corona.

Figure 1 shows the average temperature and density stratification as a function of height for Run 2 at a time shortly after the empirical coronal heating of equation (12) is activated. The visible surface is located approximately 3.1 Mm above the base of the computational domain, just below the point where superadiabatic stratification transitions into a $\sim 4100$ K temperature plateau. We note that the model atmosphere is highly dynamic, and that the average thickness of the region between the surface and the transition region boundary
(positioned approximately 5.1 Mm above the base of the domain) can vary substantially with time. This time-varying, relatively thick dynamic layer connecting the surface to the transition region is characteristic of our Quiet Sun models — the physics of an active region (particularly during emergence) can be substantially different.

As described in Section 2.1.3, empirically-based coronal heating is applied to the entire sub-volume above the surface. However, the radiative source terms still dominate the energetics of region between the surface and the transition region boundary. Thus, relatively cool average temperatures are maintained within this ~ 1500 km-thick layer. This is not to say there is an absence of hotter plasma at these heights. Since the empirical heating is deposited preferentially in places where the field is strong, this region tends to be magnetically heated only along thin, isolated magnetic structures associated with the strong concentrations of photospheric flux swept into intergranular lanes. We note that a more physical treatment of the radiation field would likely mitigate some of the excessive cooling in certain parts of our model chromosphere.

Typical deviations from the average stratification can be seen in Figure 2, where the gas density and temperature along a vertical x-z slice through the Cartesian domain are shown. The average gas pressure and internal energy as a function of height for the same timestep are shown in Figure 3, along with both the average value and range of \( \beta \). The strongest concentrations of magnetic flux at the model photosphere are of order 1 kG (corresponding to an extremal plasma-\( \beta \) value of order unity), and are correlated with the vortical, low-entropy downflows typical of convective turbulence in a stratified medium (Stein & Nordlund 2002). Except within these localized regions, the layers near the surface behave as a high-\( \beta \) plasma with the magnetic field frozen into the fluid.

In the overshoot region above the surface, where radiative source terms dominate the energetics and the super-adiabatically stratified convection zone transitions into a non-turbulent regime, fluid in the diverging upflows of convective cells is rapidly cooled, and turns over within a local pressure scale height (of order \( 10^2 \) km). Since the magnetic field remains essentially entrained in this overturning fluid, we see a precipitous drop in the horizontal average of the magnetic field strength with height just above the surface. Although there is a corresponding drop in the average gas pressure, the layers just above the Quiet Sun surface retain the characteristics of a dynamic, but non-turbulent high-\( \beta \) plasma threaded by an interconnected network of thin, isolated magnetic structures. These structures remain pressure confined in a region that extends, on average, to a height of ~ 1 Mm above the surface. The plasma-\( \beta \) is of order unity along this interconnected network, yet it remains high elsewhere as reflected by the range of \( \log \beta \) between ~ 3.2 and 4.2 Mm in Figure 3. Above the transition region boundary, the surrounding gas pressure becomes low enough to
allow the magnetic structures to expand into the low-density, high-temperature corona.

To visualize the thermodynamic fluctuations and the distribution of magnetic flux both above and below the visible surface, we display grayscale images of the gas temperature and the vertical component of the magnetic field along three horizontal slices through the computational domain: one positioned at the surface, and two others, one located 700 km below the surface in the convective interior and one positioned 700 km above the surface in the model chromosphere. Figure 4 displays these quantities for Run 1, and Figure 5 displays them for Run 2. Each snapshot represents a relaxed state. We note that the snapshot shown in Figure 5 is identical to that used to generate Figures 1-3. The lower row of each panel can be thought of as a series of synthetic, relatively high-resolution magnetograms taken at different heights in the atmosphere (the middle and right frames in the lower row of each image correspond to photospheric and chromospheric magnetograms respectively).

The global morphology of the two runs is similar in almost every respect. In each case, the convective portion of the computational domain exhibits the standard characteristics of magnetoconvection in a stratified medium: asymmetric vertical flows, strong coherent low-entropy downflows that penetrate deep into the interior, and a concentration of vertical flux at the surface into intergranular lanes and vortical downdrafts. Similarly, the surface overshoot layer and model chromosphere share a number global characteristics: First, the layers below the transition region-corona boundary behave on average as a high-$\beta$ plasma interspersed with threads of flux that serve as a magnetic connection between the “salt and pepper” concentrations of flux at the model photosphere and the coronal magnetic field. Second, the dynamic atmosphere is far from force-free, and thus may not compare favorably with force-free or potential field extrapolations anchored at the model photosphere. Third, like the simulations of Cheung et al. (2007b), each run exhibits a temperature reversal of the convective pattern with height in the chromosphere (see column 3 of Figures 4 and 5 where hotter material mirrors the relatively cool, dense intergranular lanes of the photosphere).

We note that a brightness reversal with height is a common feature of Ca II H and K observations of the Quiet Sun chromosphere (see e.g., Rutten et al. 2004 and references therein). We find that the temperature reversal in our simulations is a consequence of the interplay between the two dominant source terms of the energy equation in this region: radiative cooling, and $p \nabla \cdot \mathbf{v}$ work. Above the surface (where convective cells cool radiatively and turn over), diverging flows are associated with cell centers, while converging flows are associated with the intergranular lanes. Thus, on average, the $p \nabla \cdot \mathbf{v}$ work acts to heat the intergranular lanes and to cool the plasma above diverging granules. Although the radiative source term acts in the opposite fashion (e.g., cooling the relatively dense chromospheric plasma above intergranular lanes), and dominates the energetics at the surface, it drops off
with increasing height roughly as \( n^2 \) on average (where \( n \) is the number density). Thus, above the surface, \( p\nabla \cdot \mathbf{v} \) work tends to dominate. The net effect is that the edges of the cells are heated in the chromosphere, resulting in the temperature reversal shown in column 3 of Figures 4 and 5. We note that the magnetic heating in these layers is negligible.

The fact that the two runs are so similar is not particularly surprising — each run was relaxed so that its average stratification compared favorably to existing, more realistic models. There are however, subtle but important differences between the two approaches. As expected, the most important differences result from discrepancies in the average gas pressure throughout the layers between the visible surface and the transition region boundary. While the sub-surface and the coronal sub-domains of Run 2 behave essentially as an ideal gas, there are significant deviations near the surface, and in the layers of the upper atmosphere near the transition region boundary where the magnetized plasma transitions into a low-\( \beta \) regime.

These differences have the following consequences. First, since the photospheric gas pressure is unphysically high in Run 1, the plasma-\( \beta \) value in the kilogauss field concentrations at the surface is uncharacteristically high — a factor of 5 to 10 times greater than similar regions in Run 2. This leads to subtle, but noticeable differences in the magnetic connectivity of the lower atmosphere. Second, as is suggested by Figures 4 and 5, a typical granule of Run 2 is slightly smaller than a typical granule of Run 1. This difference is small; in fact, the average granule size seems to be more sensitive to the treatment of radiative cooling at the surface (thus the average stratification of the atmosphere) and the depth of the model convection zone than to differences in the treatment of the equation of state.

Finally, we note a secondary effect. In order to achieve a reasonable average thermodynamic stratification at and below the surface, the parameterization of the optically-thick surface cooling between the two runs must differ (as described previously). This leads to a difference in the thickness of the model chromosphere between the two runs: the transition region of Run 1 lies closer to the surface on average than it does in Run 2. This tends to flatten some of the magnetic structures in the chromospheric network of Run 1, and allows the field to expand into a low-pressure medium at a slightly lower altitude in the atmosphere (note that the flux threading the chromosphere of Run 1 looks slightly more diffuse than that of Run 2 at the same height).
3.3. The Distribution of Magnetic Flux at the Surface

A robust feature of the surface flux distribution at all times during either run is the tendency for magnetic flux to be swept into, and pushed along, narrow intergranular lanes. Since it is common for strong, oppositely-directed flux to be associated with converging flows and downdrafts, we often see evidence of flux cancellation (the apparent disappearance of oppositely directed magnetic flux at the photosphere). While a detailed analysis of flux cancellation is beyond the scope of this paper, we note that such an apparent cancellation could be the result of a number of processes (see, e.g., Welsch 2006): the advection and submergence of a small Ω-shaped magnetic structure (or the emergence of a U-shaped loop), the reconnection of oppositely-directed flux constrained within convergent flows, or the disappearance of opposite polarity flux as a result of the resolution limit of a given instrument. We address the latter process here, and ask the question: Can we properly characterize the surface evolution of magnetic flux that would be seen at a coarser resolution? In addition, it is interesting to ask the question: What average level of unsigned flux would we measure in our Quiet Sun model photosphere given the resolution limit of available instruments?

Figure 6 shows a timeseries of noise-free synthetic photospheric magnetograms generated from Run 1 at its native resolution (0.16 arc seconds), and at the approximate resolution limit of the MDI and Kitt Peak magnetographs: ∼0.62 and 1 arc second resolution respectively (the high-resolution mode of MDI). The images were generated by convolving the simulated dataset with a Gaussian response function whose FWHM corresponds to the stated resolution of each instrument. The elapsed solar time in this series is 15 minutes.

During any 15 minute interval one is likely to find one or two events that could be described as a flux cancellation. This interval is no exception, as there are several places where strong, small-scale concentrations of oppositely-directed magnetic flux (small bipoles) converge and disappear as the magnetic field entrained in the fluid gets swept below the surface by strong downdrafts or collapsing granules. Although individual cancellation events can be identified and tracked at each degraded resolution (for example, note the cancellation near the top right-hand corner of the box), it is difficult to relate the disappearance of bipoles to the physics of the convective flow pattern, and thus it is difficult to interpret the physics of this apparent cancellation at these coarser resolutions.

In general, we find that emergence events appear over a slightly larger area and appear somewhat more diffuse in the images than cancellation events. This reflects the fact that most of the magnetic field is entrained in the overturning plasma and emerges near cell centers, gets advected into the intergranular lanes, and submerges near highly-concentrated vortical downflows. It is also conceivable that the coalescence of magnetic flux between granules could be interpreted in a degraded image as an emergence event, or as the sudden
appearance of a unipolar flux element as the magnetic field strength in a localized region becomes more concentrated (see Lamb et al. 2007, and the discussion therein).

The simulations show that often, magnetic flux residing in the intergranular lanes consists of opposite polarities positioned nearly adjacent to one another (as is apparent in the top row of Figure 6). If resolution is coarse, the observed flux per pixel area is essentially an average over the positive and negatively-directed flux; this may lead to an underestimate of quantities such as the total unsigned flux in the region. To demonstrate this effect, we calculate the average unsigned flux per pixel for the three snapshots at each resolution. The average unsigned flux per unit area in the simulated dataset is 34.5 G. Performing the same calculation at MDI resolution yields 19.9 G, while at Kitt Peak resolution we obtain only 15.0 G. (we note that the Pevtsov et al. 2003 observed value is 5.5 G per Kitt Peak pixel for the Quiet Sun). Thus, it is likely that a relatively low-resolution magnetograph can significantly underestimate the amount of unsigned magnetic flux present in the Quiet Sun photosphere.

3.4. The Magnetic Structure of the Quiet Sun Model Atmosphere

To better visualize the magnetic structure of the atmosphere, in the top panel of Figure 7 we display \( \log B \) along a vertical \( y-z \) slice through a portion of the domain that includes both the corona, and the layers just below the visible surface. The white line denotes the transition region lower boundary, and the dark line indicates the location of the visible surface. We determine these boundaries by first defining the location of the visible surface and transition region boundary in the initial average background stratification (as described in Section 3.2). Then, for each horizontal pixel at later times we determine the lowest height for which this value is achieved (interpolating vertically between cells as necessary).

The vertical slice shown in the top and bottom panels of Figure 7 illustrates the different environments present in the upper atmosphere and corona; its position was chosen so as to pass through portions of the volume that contained both relatively strong, and relatively weak coronal magnetic fields. The position of the vertical plane is overlaid as a vertical white line on the three images in the center row of Figure 7. These images represent (from left to right) contours of \( \log \beta \), the magnetic flux, and \( \log B \) along the two-dimensional surface representing the lower boundary of the transition region (corresponding to the horizontal white line in the top and bottom rows). The bottom image of Figure 7 is an image representing the logarithm of the current density along the same vertical slice as shown above.

The top image of Figure 7 shows that the magnetic connection between the photospheric and coronal magnetic field is highly complex, and lends itself to a simple, canopy-like inter-
pretation only around regions where the field is most concentrated (e.g., note the relatively continuous expanding plume in the strong field region near the left border of the top panel of Figure 7). The complexity of the atmosphere is again reflected in the lower frame of Figure 7 (and in Figure 8) where it is evident that the Quiet Sun chromosphere is far from potential (current-free). In addition, we find that the bulk of the magnetic energy is concentrated at and below the surface, and that there is a persistent current layer corresponding to a local peak in the average magnetic field strength near the transition region boundary (this peak is also reflected in the average value of \( \log \beta \) around \( z = 5.5 \text{ Mm} \) in Figure 3). It is at these heights that magnetic structures in the dynamic chromosphere can enter and expand into the low pressure, stable corona. Here, the magnetized plasma of isolated magnetic structures transitions between a \( \beta \sim 1 \) to a low-\( \beta \), stable environment, and on average, once this flux penetrates into the corona, it tends not to be readily recirculated back down into the lower atmosphere.

Figure 8 shows the logarithm of the magnitude of the Lorentz force at a later timestep in Run 2, \( \log |\mathbf{F}_L| = \log |\mathbf{J} \times \mathbf{B}| \) (second column), along with a normalized measure of the degree to which the current is aligned with the magnetic field in the photosphere, \( |\cos \theta| = |\mathbf{J} \cdot \mathbf{B}|/(|\mathbf{J}| |\mathbf{B}|) \) (third column). For reference, the gas temperature is also shown (first column). As \( \cos \theta \) approaches unity, the angle \( \theta \) between \( \mathbf{J} \) and \( \mathbf{B} \) is reduced, the current becomes field-aligned, and the atmosphere can be considered force-free (\( \mathbf{J} \times \mathbf{B} = 0 \)). If this is the case, it is possible to use static extrapolation techniques to study the magnetic topology of the atmosphere (see Schrijver et al. 2006 and references therein). However, column 3 of Figure 8 indicates that the low atmosphere deviates significantly from a force-free configuration, and that \( \theta \) itself exhibits a significant amount of fine structure at scales down to the level of resolution of the run. This structure is mirrored in the force-free parameter \( \alpha \) (defined by the relation \( \nabla \times \mathbf{B} = \alpha \mathbf{B} \), from which follows \( \alpha = |\mathbf{J} \cdot \mathbf{B}|/B^2 \)). We note that while similarly complex, the structure of \( \alpha_z = J_z/B_z \) can differ substantially from that of \( \alpha \) itself. The \( \alpha \) distribution becomes smoother (less structured) and the atmosphere becomes somewhat more force-free only in the layers above the transition region boundary. This is not to say these layers are current-free; the dynamic upper atmosphere still deviates significantly from a potential field configuration. In addition, we find that the horizontally averaged current helicity density (\( \mathbf{J} \cdot \mathbf{B} \)) along each layer in the atmosphere (and below) is essentially zero in Runs 1 and 2, though there can be significant buildups of helicity density locally near vortical flows. Thus, over time, in the absence of non-inertial Coriolis forces, the convectively unstable layers do not introduce a net current helicity into the model atmosphere, as expected.

To get a better idea of the magnetic structure of the low atmosphere, we draw several hundred fieldlines anchored at evenly-spaced points along equidistant horizontal lines on a
plane positioned $\sim 100$ km below the visible surface. In Figure 9, we superimpose these fieldlines onto an image of the vertical velocity at the surface in order to visualize the magnetic connectivity relative to the granulation pattern. Since the fieldlines originate from below the photosphere, most turn over within a few local pressure scale heights. Many form tiny closed loops straddling intergranular lanes (the lowest-lying loops evident in Figure 9), but many more in this sample are seen to form extended connections with concentrations of magnetic flux within the lanes surrounding other convective cells. In fact, it is quite common in the simulations to see long, horizontal ribbons of low-lying flux threading the low chromosphere. Of the fieldlines that extend into the corona, most are oriented vertically (reminiscent of spicule or fibril-like structures). This subset of fieldlines is most evident in the lower half of Figure 9 where the vertical fieldlines appear to end abruptly. This reflects the fact that magnetic flux is allowed to penetrate the upper coronal boundary of the simulation domain.

To investigate the interaction of magnetic fields generated at and below the surface with pre-existing magnetic configurations in the corona (e.g., due to distant active regions, or other global magnetic structures in the corona), we add a simple, weak background field (relative to the average magnetic field strength below the surface) of 3 G to Run 2: in one run, we orient the additional field vertically, and in a second run we orient the additional field in the horizontal direction. We note that in the first case, we must open the lower boundary to allow magnetic flux to penetrate through the convective layers. We implement this by imposing a simple zero-gradient boundary condition along with an additional source term to the vertical component of the momentum equation in the layers at and just above the lower boundary that acts to ensure a zero net mass flux through the lower penetrative boundary. In each of the two cases, we allow the simulations to progress for several additional convective turnover times.

Among the most noticeable features of these runs are the persistent horizontally-directed ribbons of magnetic flux that permeate the model chromosphere. These features are a common morphological characteristic of all of our simulations, and are apparent in Figure 10 (where we show the interaction of previously generated Quiet Sun fields with a weak vertical coronal field) as both twisted and untwisted magnetic structures near the transition region boundary. In this figure, a subset of magnetic fieldlines are anchored at the upper coronal boundary, and we trace the magnetic connectivity of the fields permeating the model corona to their more concentrated footpoints at and below the visible surface. This snapshot reveals how localized convective downdrafts can affect relatively strong concentrations of magnetic flux at and above the photosphere. In particular, the image shows how concentrations of horizontally-directed magnetic field entrained in overturning plasma above the surface can become caught in a strong sub-photospheric convective downflow, effectively pushing
magnetic flux back through the surface into the convective interior. In a photospheric magnetogram, this process can result in the appearance of a (generally weak) bipole, when in fact, no new magnetic flux has emerged from below the surface.

Figure 11 displays ~1000 fieldlines initiated from a grid of points positioned in the low chromosphere ~400 km above the surface. The fieldlines are extended both upward into the model corona, and downward through the photosphere into the model convection zone. This snapshot is taken from one of the later timesteps of a run where we simulate the interaction of dynamo-generated fields with an initially weak horizontal magnetic field. Here, we position the viewing angle to look directly down on the visible surface from above. A consequence of the mixed-\(\beta\) dynamic environment of the chromosphere is the complex connectivity between magnetic structures. Again, flux concentrations at the photosphere are magnetically connected to rather distant regions of the atmosphere via horizontal magnetic structures threading the low chromosphere. In general, this dynamic behavior cannot be reproduced by static extrapolations generated from photospheric magnetograms. Thus, to the extent that the Quiet Sun corona can be described as force-free, extrapolations anchored in the upper chromosphere may yield more accurate representations of the Quiet Sun field than those anchored at the photosphere.

### 3.5. Resistive and Viscous Heating of the Quiet Sun Model Corona

Finally, we turn our attention to the coronal heating mechanism of the code. Up until now, we maintained a hot 1 MK corona in our ideal MHD calculations using the empirically-based energy source term of equation (12). We now de-activate this source, activate the viscous and resistive source terms in our momentum, induction, and energy equations (equations [2]-[4] respectively), and determine whether it is possible to generate or maintain a hot corona in the absence of empirically-based magnetic heating. We set the coefficients of kinematic viscosity and magnetic diffusivity to empirically-obtained values that exceed by a small amount the numerical viscosity and resistivity characteristic of the code at its current resolution. We then initiate two short runs. The first uses as its initial state a snapshot from the relaxation process of Run 1 at a time when the corona has yet to heat up, and the second uses an initial state with a hot corona taken from the end of Run 2.

Admittedly, our numerical resolution could be improved around current sheets, and we have yet to experiment with more sophisticated, depth-dependent or current-dependent coefficients of magnetic resistivity (see e.g., Chen & Shibata 2000; Otto 2001). Nevertheless, it is interesting to report that our current implementation of resistive heating did not provide the necessary energy in our Quiet Sun simulations to generate or maintain a corona at X-
ray emitting temperatures (see Figure 12, which shows how quickly the corona cools in the absence of the empirical source term). Simply put, the bulk of the Joule heating is deposited in regions of the lower atmosphere dominated by radiative processes. Even in the portion of the domain representing the transition region and corona, the energy balance is dominated by the effects of optically-thin radiative cooling and thermal conduction — on average, the effects of resistive dissipation and viscous heating are negligible. Thus, within the assumptions and limitations of our model (not the least of which is the limited spatial extent of the portion of the domain representing the corona), we must include an additional empirical heating function in order to maintain a model corona at observed temperatures.

4. Discussion and Conclusions

We have presented a set of three-dimensional MHD simulations of the Quiet Sun magnetic field within a single computational domain that extends from the upper convection zone, through the photosphere and chromosphere, and into the transition region and low corona. We include, within the system of MHD equations, much of the important physics that is believed to govern the evolution of these diverse parts of the solar atmosphere. These simulations are the first of their kind, and the numerical techniques presented here can be used to bridge the gap between sub-surface calculations (e.g., Fan et al. 1999; Abbett et al. 2000) and coronal models (e.g., Mok et al. 2005; Welsch et al. 2005) in a physically self-consistent way.

It is important to recognize, however, that there are significant limitations to our current approach. The most obvious is the current parameterization of optically-thick radiative transfer in the model photosphere and chromosphere. While our simplified treatment has the advantage of being computationally efficient, we are limited in our ability to directly compare our simulations with observational data, and thus must be careful not to over-interpret our results. Since we do not solve the radiative transfer equation in detail, we must rely on more sophisticated models of surface convection (e.g., Bercik 2002) to provide the information necessary to calibrate our approximate treatment and achieve a sufficiently solar-like average thermodynamic stratification. We are currently in the process of improving our treatment by incorporating the LTE radiative transfer equation (in the three-dimensional Eddington approximation as described by Unno & Spiegel 1966) into the implicit sub-step of RADMHD. Once completed, this will provide a more physical basis for surface cooling in both Quiet Sun and active region model atmospheres. We hope to report on the results of this effort in the near future.

There are a number of additional limitations that should be addressed. First, the sim-
ulations presented here are of relatively modest resolution, and second, cover a relatively limited area. The former restriction affects our ability to resolve current sheets and other small scale structures, and may impact the efficacy of Joule heating in these localized regions. The latter impacts our ability to model extended, large-scale magnetic features in the upper atmosphere. In particular, the proximity of the upper open boundary to the visible surface may significantly impact the global structure of the atmosphere. For example, once large-scale twisted loops emerge from below the surface, they rapidly encounter the upper boundary. Once this occurs, the magnetic connection between loop footpoints in the lower atmosphere is severed. Magnetic tension that would have normally been present in a closed loop is lost, as helicity is transported out of the domain. This can impact the magnetic topology of the atmosphere as a whole. Of course, this is a practical limitation that can be directly addressed with additional computational resources.

In addition, the thermodynamic structure of the upper atmosphere is sensitive to the particular choice of weighting function for our empirically-based coronal heating mechanism (see equation [11]). While a thorough parameter space survey can help determine a weighting function that better reproduces characteristic transition region and coronal emission, a better physical understanding of the coronal heating mechanism is ultimately required.

The simulations presented in this paper represent only a first step in our effort to understand the origin and evolution of the Quiet Sun magnetic field — clearly, further study is required (e.g., systematic studies of Quiet Sun magnetic fields at higher resolution and over much larger spatial scales than presented here, and detailed studies of the decay of active region magnetic fields). However, with the above limitations of our computational models in mind, we are able to draw some general conclusions based on our simulations to date:

- If we use an empirically based coronal heating mechanism consistent with the Pevtsov et al. (2003) relationship between X-ray emission and magnetic flux observed at the surface, magnetic fields generated from a convective dynamo are sufficient to heat the corona to 1 MK.

- If a non current-dependent, constant coefficient of magnetic resistivity is used, Joule heating in our modest-resolution Quiet Sun model atmosphere is insufficient to maintain a sufficiently hot corona in the absence of any additional empirically-based source term.

- A non-ideal equation of state is necessary to obtain more realistic ratios of magnetic to gas pressure at, and just above, the model photosphere. However, an ideal treatment is capable of reproducing many of the global characteristics of the non-ideal atmospheres.
On average, the bulk of the unsigned magnetic flux resides below the visible surface. Of the flux that threads the photosphere, most remains entrained in the plasma, and turns over within a local pressure scale height. Thus, there is a steep decline in the average amount of unsigned magnetic flux with height above the surface.

The model chromosphere exhibits a reverse granulation pattern — plasma above cell centers tends to be cooler than the average temperature at that height, while plasma above the photospheric intergranular lanes is generally hotter. In the models, this is simply due to the $p\nabla \cdot \mathbf{v}$ work of converging and diverging flows in the relatively low-density layers above the photosphere where the radiative source terms are less dominant.

The magnetic connectivity in the dynamic region between the visible surface and upper transition region is complex. This region is characterized by the presence of both twisted and non-twisted $\beta \sim 1$ horizontally-directed magnetic structures that thread the chromosphere and connect relatively distant concentrations of magnetic flux in the photosphere with the transition region footpoints of coronal structures.

Synthetic vector magnetograms generated in the model photosphere and chromosphere are substantively different, and will yield substantively different results if used as a basis for force-free or potential field extrapolations. Neither extrapolation technique will reproduce the twisted horizontal structures present just above the model photosphere (and their associated magnetic connectivity) in the dynamic, often non-force-free layers of the low chromosphere.

Analysis of magnetograph data at relatively low, one arcsecond resolution can lead to a significant underestimate of the total amount of unsigned magnetic flux threading the Quiet Sun photosphere.

A persistent current layer is formed as the atmosphere transitions from a dynamic, high-$\beta$ regime (on average) to the more magnetically dominated transition region and corona. The Quiet Sun model corona is dynamic, and is not everywhere force-free or even low-$\beta$. There are regions of both relatively strong and exceptionally weak field high in the model corona.

A magnetic carpet is generated and maintained through the action of a convective dynamo without first requiring the dispersal of a model active region.

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Fig. 1.— The horizontally-averaged temperature and density stratification for Run 2 showing the existence of a hot corona during a time when the empirically-based magnetic heating is active. The cool average temperature just below the transition region boundary (indicated by the thin vertical line on the right) reflects the localized nature of the empirically-based heating function (heating is deposited preferentially along concentrations of magnetic flux threading the model chromosphere), and the dominance of radiative cooling in these layers. The thin vertical line on the left indicates the position of the visible surface.
Fig. 2.— The density (left) and temperature (right) stratification along a single vertical ($x$-$z$) slice through the computational domain. These surface plots show typical deviations from the average background stratification in the dynamic model atmosphere. Note the difference in scale between the $x$ and $z$ axes.
Fig. 3.— An illustration of the large variation in the Quiet Sun plasma-β throughout the computational domain. The figure shows the horizontally-averaged value of log β as a function of height at the same timestep shown in Figures 1 and 2, along with the range of values present at a given height (vertical lines). Also shown are the horizontally-averaged gas pressure and internal energy per unit volume as a function of height. Note that there are relatively significant deviations from an ideal gas equation of state near the visible surface (z ∼ 3.1 Mm) and at the transition region boundary (z ∼ 5.1 Mm).
Fig. 4.— The temperature variations characteristic of a Quiet Sun model using an ideal equation of state. **Top row:** The gas temperature along a horizontal slice through the \(30 \times 30 \times 7.5\) Mm\(^3\) domain of Run 1 at 700 km below the photosphere (left), at the surface (center), and 700 km above the surface (right). Dark (light) regions indicate temperatures cooler (hotter) than average. **Bottom row:** The vertical component of the magnetic field along the same horizontal slices. Dark (light) regions indicate the presence of magnetic flux directed away from (toward) the observer. For this particular snapshot, the maximum value of \(|B_z|\) threading the photosphere in the intergranular lanes and strong downdrafts is 1.2 kG.
Fig. 5.— The temperature variations characteristic of a Quiet Sun model using the non-ideal, OPAL equation of state. The panels show the same quantities as Figure 4; this time, for Run 2.
Fig. 6.— An illustration of the effects of degraded resolution on synthetically-generated magnetograms. **Top row:** The magnetic flux threading the surface during a time interval when several flux cancellation events occurred: $t = t_0$ (left), $t = t_0 + 7.5$ minutes (center), and $t = t_0 + 15$ minutes (right). **Middle row:** Simulated noise-free magnetograms of the same three snapshots at MDI resolution of $\sim 0.62$ arc seconds (high-resolution mode). **Bottom row:** The same sequence at Kitt Peak resolution ($\sim 1$ arc second resolution). The average unsigned flux per pixel (averaged in time over the three snapshots) is 34.5 G; at Kitt Peak resolution this reduced to 15.0 G. The grayscale intensity is normalized between the three images at each resolution.
Fig. 7.— A visualization of the complex magnetic structure of the Quiet Sun model atmosphere. **Top:** The logarithm of the magnetic field strength along a vertical \((y-z)\) slice through a portion of the domain (deeper layers below the surface are excluded). The white line indicates the position of the transition region lower boundary, and the black line denotes the position of the visible surface. **Middle left:** Grayscale image of \(\log \beta\) along a two-dimensional surface representing the transition region boundary (corresponding to the horizontal white line in the top and bottom rows). The vertical white line represents the position of the vertical slice displayed in the top and bottom images. **Middle center:** The vertical component of the magnetic field along the same surface. **Middle right:** The logarithm of the magnetic field strength along the same surface. **Bottom:** The logarithm of the current density along the same vertical slice as the top image. The left-hand side of the top and bottom image corresponds to the top of the vertical line in each of the middle three images.
Fig. 8.— A visualization of the extent to which the Quiet Sun model photosphere and chromosphere can be considered force-free. **Top Row:** The gas temperature along a horizontal slice positioned at the visible surface (left), the logarithm of the magnitude of the Lorentz force, (log |J × B|) along that same slice (middle), and a normalized measure of the degree to which the current is aligned with the magnetic field at the surface, |cos θ| = |J · B|/(|J||B|) (right). White represents |cos θ| = 1 (field-aligned, and thus force-free) and black represents |cos θ| = 0. **Bottom Row:** Corresponding quantities ~ 450 km above the surface in the low chromosphere.
Fig. 9.— The magnetic configuration of the low atmosphere showing the many low-lying, horizontally-directed magnetic structures typical of the Quiet Sun model atmosphere. Specifically, we show a set of magnetic fieldlines that penetrate the model photosphere from below: the fieldlines were generated from evenly-spaced points along equidistant horizontal lines on a plane positioned ∼ 100 km below the visible surface. The grayscale image represents the vertical component of the velocity field along this plane. Only a portion of the computational domain is shown.
Fig. 10.— An illustration of the complex magnetic connectivity of the model atmosphere as Quiet Sun magnetic fields generated by a convective dynamo interact with a weak, initially vertical coronal field. Shown are magnetic field lines initiated from equidistant points along a horizontal line positioned near the upper boundary of the model corona. The image shows how magnetic flux entrained in overturning flows and strong convective downdrafts can be pushed below the surface. The horizontal slice denotes the approximate position of the visible surface, and grayscale contours of vertically directed flows (dark shades indicate downflows, light shades indicate upflows) are displayed along the slice. Only a portion of the domain is shown. Inset images: A timeseries (over ~ 5 minutes) of the magnetic flux penetrating a small portion of the model photosphere. This sub-domain is centered on the location featured in the background image where magnetic flux is being advected below the surface.
Fig. 11.— The magnetic structure of the chromosphere for a run where dynamo-generated Quiet Sun magnetic fields have interacted with an initially weak horizontal coronal field for some time. Shown are magnetic field lines initiated from a set of points located in the low chromosphere ∼ 400 km above the visible surface. Note how the field remains essentially frozen into the overturning plasma above granules. The grayscale intensity on the horizontal slice denotes the magnitude of the vertical velocity at the surface. Only a portion of the domain is shown.
Fig. 12.— The horizontally-averaged temperature stratification during a time when the empirically-based magnetic heating is active (solid line), and 4.5 minutes after the empirically-based coronal heating is shut down leaving resistive dissipation as the principal coronal heating mechanism (dashed line). The figure demonstrates that Joule heating alone over this time period in our model was not sufficient to maintain a corona at X-ray emitting temperatures.
Table 1. Envelope function parameters for Runs 1 and 2: steepness parameters \(a_i\) and density or position cutoffs \((\rho_i, z_i)\) for the energy source terms (cgs units). See Section 2.1 for details.

<table>
<thead>
<tr>
<th>Run</th>
<th>(\rho_1)</th>
<th>(z_{2l})</th>
<th>(z_{2u})</th>
<th>(\rho_3)</th>
<th>(\rho_4)</th>
<th>(a_1)</th>
<th>(a_{2l})</th>
<th>(a_{2u})</th>
<th>(a_3)</th>
<th>(a_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(9.75 \times 10^{-7})</td>
<td>n/a†</td>
<td>n/a†</td>
<td>(1.53 \times 10^{-5})</td>
<td>(2.15 \times 10^{-13})</td>
<td>(3/\rho_1)</td>
<td>n/a†</td>
<td>n/a†</td>
<td>(10/\rho_3)</td>
<td>(3/\rho_4)</td>
</tr>
<tr>
<td>2</td>
<td>(4.50 \times 10^{-8})</td>
<td>(3.09 \times 10^8)</td>
<td>(3.45 \times 10^8)</td>
<td>(9.01 \times 10^{-6})</td>
<td>(4.50 \times 10^{-13})</td>
<td>(3/\rho_1)</td>
<td>(50/z_{2l})</td>
<td>(50/z_{2u})</td>
<td>(10/\rho_3)</td>
<td>(3/\rho_4)</td>
</tr>
</tbody>
</table>

†\(\xi_2 = 0\) for Run 1