Convective Dynamos and the Minimum X-ray Flux in Solar-Type Main Sequence Stars

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Abstract:

We investigate whether a small-scale convective dynamo can account quantitatively for the observed lower limit of X-ray surface flux in solar-type main sequence stars. Our approach is to use 3D numerical simulations of a turbulent dynamo driven by convection to characterize the dynamic behavior, magnetic field strengths, and filling factors in a non-rotating stratified medium, and to predict these magnetic properties at the surface of cool stars. We use simple applications of stellar structure theory for the convective envelopes of main-sequence stars to scale our simulations to the outer layers of stars in the F0-M0 spectral range, which allows us to estimate the unsigned
magnetic flux on the surface of non-rotating reference stars.

With these estimates we use the observed magnetic flux-X-ray flux correlation of Pevtsov et al. (2003) to predict the level of X-ray emission from such a turbulent dynamo, and find that our results compare well with observed lower limits of surface X-ray flux. This suggests that dynamo action from a convecting, non-rotating plasma is a viable alternative to acoustic heating models as an explanation for the basal emission level seen in chromospheric, transition region, and coronal diagnostics from late-type stars.
Background:

- Understanding the origin of magnetic activity in stars has been an important research area in astronomy and astrophysics for many years.
  - Imaged solar observations show a clear link between magnetic fields and the formation of heated plasma in the chromosphere, transition region and corona, especially in active regions (see Fisher et al. 1998).
  - Active regions are believed to form from loops of magnetic flux that emerge from the base of the solar convection zone (see, e.g., Fan 2004). The large-scale field on the Sun is believed to originate via a global-scale dynamo, the $\alpha-\Omega$ dynamo, powered mainly by velocity shear (differential rotation).
  - It is often assumed that the amount of differential rotation increases as the rotation rate itself increases.
- Skumanich (1972) was among the first to propose an observational connection between rotation rate and the level of activity in stars through dynamo action. Noyes et al. (1984) went on to show a correlation between Ca II H+K surface flux and Rossby number.

- For cool stars, a straightforward application of the rotation-activity relationship leads one to conclude that slowly- or non-rotating stars should show little or no activity diagnostics. However, this is not the case.
  - There appears to be a lower limit to the emission of activity indicators in the chromosphere, transition region and corona.
  - Given the expected absence of a global dynamo in non-rotating stars, this heating is usually attributed to a “basal” non-magnetic mechanism such as acoustic waves.
The role of basal flux in observed X-ray emission is more uncertain compared to chromospheric and transition region diagnostics.

- Rutten et al. (1991) did find a strongly color dependent lower bound on the X-ray emission of stars in his data set. However, it was a magnitude-limited sample and included giant stars.

- Schmitt (1997) clearly shows that the lower bound on X-ray emission in cool dwarf stars has a weak color dependence at best; which, if it exists is in the opposite sense found by Rutten et al. (1991). The lower bound found was log F_X ~ 3.7 (in cgs units).

- The volume-limited sample of Schmitt (1997) showed that essentially all stars with outer convection zones emit X-rays with characteristics similar to solar coronal emission. It seems, therefore, that all “cool stars” possess coronae.
Recent work analyzing both solar and stellar data (Pevtsov et al. 2003) indicates a clear and unambiguous relationship between unsigned magnetic flux and coronal X-ray emission that extends over twelve orders of magnitude.

- This study lends credence to the idea that at least for X-ray emission, a “basal” heating mechanism may indeed be magnetic in origin, but unlike the “excess” emission from global fields, the magnetic flux has little or no connection to stellar rotation.
- High resolution observations of the Sun indicate a viable magnetic origin for this “basal” magnetic activity emission: the existence of a small-scale mixed polarity magnetic field component which appears to be independent of the solar cycle.
• Complementing the observational evidence for small-scale magnetic fields is theoretical research into “fast dynamos” (e.g., Vainshtein et al. 1996; Tanner & Hughes 2003)
  – Breakthrough made by Cattaneo (1999), demonstrated via a Boussinesq 3D MHD simulation that a small seed field embedded in a turbulently convecting, highly conducting plasma can indeed grow exponentially until the magnetic energy reaches 10-20% of the kinetic energy of the convective motions.

• These factors lead to the question: Is the turbulent convective dynamo a viable mechanism to generate enough X-ray flux to account for the observational limit?
  – To address this question, we extend the investigation of field generation by a convective dynamo beyond the Boussinesq approximation to the “anelastic” regime which allows for gravitational stratification and use these results to predict the level of X-ray emission in main sequence stars.
Description of the Model Dynamo

- We solve a non-dimensional form of the 3D anelastic MHD equations in a horizontally periodic, vertically closed Cartesian domain (see Lantz & Fan 1999; Fan et al. 1999)

- The resolution of the non-rotating box is $288 \times 288 \times 72$ and spans 5 pressure scale heights and 3 density scale heights vertically

- The Reynolds number and Prandtl number are defined at the bottom of the domain with $R_e=750$ and $P_r=1$, and a constant dynamic viscosity is assumed
• We initiate convection by introducing small random entropy fluctuations to a state with a prescribed background entropy gradient and allow the new state to relax dynamically and thermally.

• We then set the magnetic Reynolds number $R_m = 1000$, and introduce a small dynamically unimportant magnetic seed field to the simulation.

• The boundary conditions on the magnetic field are stress-free and non-penetrating at the bottom, and the field at the upper boundary is assumed to be potential.
The seed field grows self-consistently and evolves within the computational domain as the run progresses, showing an exponential growth phase followed by a saturation phase.

Left: Temporal evolution of the domain-integrated total kinetic ($E_k$), thermal ($E_{th}$), and magnetic ($E_B$) energy fluctuations normalized to the sum of the three ($E_T = E_k + E_{th} + E_B$).
Right: Time evolution of the energy fluctuations on a linear scale. As the magnetic energy increases, the kinetic energy decreases, and the thermal fluctuations of the plasma increase.

The magnetic energy fully saturates to a time averaged value of 6.7% of the total kinetic energy.
Left: A volume rendering of the entropy perturbations (bottom frame) and the magnetic field strength $|\mathbf{B}|$ well after the field has saturated.

Strong magnetic fields are concentrated in the narrow, low-entropy downdrafts characteristic of stratified convection, particularly in the upper half of the domain.

A greater proportion of the total unsigned magnetic flux resides in the lower half of the box.
Bottom: Slices of the vertical component of the magnetic field near the top of the simulation box (left panel) and near the base of the box (right panel).

Stronger fields are concentrated and highly localized, whereas weaker field is more evenly distributed, particularly in the lower half of the box.
Connecting the Dynamo Model to the Stellar Envelope

• To determine a reference star’s surface magnetic flux, we need to scale the simulation results to physical values
  – Need to connect the velocity and magnetic fields of the simulation to the luminosity ($L$), surface gravity ($g$), radius ($R$), effective temperature ($T_{\text{surf}}$) and surface density ($\rho_{\text{surf}}$) of main sequence reference stars

• We assume that the entire stellar luminosity is carried by the convective flux use the mixing length formalism of Mihalas (1978) to scale the velocity

$$F_{\text{conv}} = \frac{L}{4 \pi R^2} = \frac{10}{\alpha} \rho V_{\text{conv}}^3$$
• We use $\alpha = 1.5$ for the mixing length parameter, as that value has been shown to reasonably match the velocity amplitude in realistic surface convection simulations (Abbett et al. 1997).

• The resulting scalings for velocity and magnetic field are (with “surf” representing the top of the domain, and “ref” representing the bottom):

$$v_{\text{surf}} = \left( \frac{\alpha L_{\text{c}}}{40 \pi \rho_{\text{surf}} R_{\text{c}}^2} \right)^{\frac{1}{3}}$$

$$B_{\text{ref}} = \left[ \frac{\pi}{5} \frac{\alpha L_{\text{c}}}{R_{\text{c}}^2} \rho_{\text{surf}}^{-\frac{1}{6}} \frac{T_{\text{ref}}}{T_{\text{surf}}} \right]^{\frac{1}{4}}$$
These scaling relationships allow use to estimate the amount of magnetic flux near the top of the dynamo simulation for given stellar parameters. The table below lists the parameters used.

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>$T_{\text{eff}}$ [K]</th>
<th>$L_*$ [erg s$^{-1}$]</th>
<th>$R_*$ [cm]</th>
<th>log $g$ [cm s$^{-2}$]</th>
<th>$\rho_{\text{surf}}$ [g cm$^{-3}$]</th>
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</thead>
<tbody>
<tr>
<td>F0</td>
<td>6949</td>
<td>$1.84 \times 10^{34}$</td>
<td>$1.05 \times 10^{11}$</td>
<td>4.27</td>
<td>$6.71 \times 10^{-8}$</td>
</tr>
<tr>
<td>F5</td>
<td>6445</td>
<td>$1.04 \times 10^{34}$</td>
<td>$9.19 \times 10^{10}$</td>
<td>4.32</td>
<td>$1.18 \times 10^{-7}$</td>
</tr>
<tr>
<td>G0</td>
<td>5948</td>
<td>$5.04 \times 10^{33}$</td>
<td>$7.52 \times 10^{10}$</td>
<td>4.42</td>
<td>$1.97 \times 10^{-7}$</td>
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<td>G5</td>
<td>5678</td>
<td>$3.31 \times 10^{33}$</td>
<td>$6.68 \times 10^{10}$</td>
<td>4.46</td>
<td>$2.42 \times 10^{-7}$</td>
</tr>
<tr>
<td>K0</td>
<td>5273</td>
<td>$1.75 \times 10^{33}$</td>
<td>$5.64 \times 10^{10}$</td>
<td>4.53</td>
<td>$3.02 \times 10^{-7}$</td>
</tr>
<tr>
<td>K5</td>
<td>4557</td>
<td>$6.88 \times 10^{32}$</td>
<td>$4.73 \times 10^{10}$</td>
<td>4.60</td>
<td>$5.33 \times 10^{-7}$</td>
</tr>
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<td>M0</td>
<td>3800</td>
<td>$2.77 \times 10^{32}$</td>
<td>$4.32 \times 10^{10}$</td>
<td>4.65</td>
<td>$9.60 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

All stellar parameters except the surface density are taken from Gray (1992) for spectral types F0-K5; those for M0 are taken from Reid & Hawley (2000). The surface densities are derived from model atmospheres of Kurucz (1993).
Results:

- The results of applying the $\alpha = 1.5$ mixing length scaling to the simulation data are presented in the table below.

The total magnetic fluxes, $\Phi_*$, were obtained by first time-averaging the total unsigned flux through a horizontal layer near the top of the simulation box and then scaled by the ratio of the surface area of the reference stars to the horizontal simulation area.

<table>
<thead>
<tr>
<th>Spectral Type</th>
<th>$H_{\text{ref}}$ [Mm]</th>
<th>$v_{\text{surf}}$ [km s$^{-1}$]</th>
<th>$\Phi_*$ [Mx]</th>
<th>$L_X$ [erg s$^{-1}$]</th>
<th>$\log F_X$ [erg s$^{-1}$ cm$^{-2}$]</th>
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</thead>
<tbody>
<tr>
<td>F0</td>
<td>4.86</td>
<td>6.67</td>
<td>$6.01 \times 10^{23}$</td>
<td>$1.86 \times 10^{27}$</td>
<td>4.13</td>
</tr>
<tr>
<td>F5</td>
<td>4.02</td>
<td>4.99</td>
<td>$4.57 \times 10^{23}$</td>
<td>$1.36 \times 10^{27}$</td>
<td>4.11</td>
</tr>
<tr>
<td>G0</td>
<td>2.95</td>
<td>3.78</td>
<td>$2.99 \times 10^{23}$</td>
<td>$8.34 \times 10^{26}$</td>
<td>4.07</td>
</tr>
<tr>
<td>G5</td>
<td>2.56</td>
<td>3.32</td>
<td>$2.30 \times 10^{23}$</td>
<td>$6.16 \times 10^{26}$</td>
<td>4.04</td>
</tr>
<tr>
<td>K0</td>
<td>2.03</td>
<td>2.79</td>
<td>$1.54 \times 10^{23}$</td>
<td>$3.88 \times 10^{26}$</td>
<td>3.99</td>
</tr>
<tr>
<td>K5</td>
<td>1.49</td>
<td>1.90</td>
<td>$9.81 \times 10^{22}$</td>
<td>$2.31 \times 10^{26}$</td>
<td>3.92</td>
</tr>
<tr>
<td>M0</td>
<td>1.11</td>
<td>1.23</td>
<td>$7.08 \times 10^{22}$</td>
<td>$1.59 \times 10^{26}$</td>
<td>3.83</td>
</tr>
</tbody>
</table>
• We estimate the X-ray luminosities of the reference stars through an empirical relationship X-ray luminosity and unsigned magnetic flux (Pevtsov et al. 2003). A fit to the data in Figure 1 of that paper leads to the relation

\[ L_X = 0.8940 \Phi^{1.1488} \]

• The resulting surface X-ray fluxes are then compared to the X-ray fluxes from a volume limited sample of cool stars (Schmitt et al. 1995; Schmitt 1997). The spectral types and absolute visual magnitudes are converted to B-V colors by calibrations in Gray (1992) and Reid & Hawley (2000).
The filled circles represent the surface X-ray flux for each observed star as a function of B-V color. The asterisks represent our theoretical prediction for the lower bound of the surface X-ray flux for $\alpha = 1.5$. The gray shaded area indicates the amount these flux levels change if the assumed surface velocities change by a factor of two.
• It is reasonable to assume that the heating mechanism that produces X-ray also produces emission in chromospheric and transition region diagnostics, contributing to the basal emission observed in these lines.

• Without a detailed magnetic heating model, we can only explore this issue using flux-flux relationships between X-ray emission and diagnostics such as Mg II emission. An often used relation is based on Table 4 of Rutten et al. (1991):

\[
\log F_{Mg\,II} = \frac{\log F_X}{1.97} + \frac{6.7}{1.97}
\]

• We compare the scaled simulation results against a survey of dK and dM stars from Mathioudakis & Doyle (1992)
The filled circles represent the surface Mg II flux for each observed star as a function of B-V color. The asterisks represent our theoretical prediction for the lower bound for $\alpha = 1.5$. The gray shaded area indicates the amount these flux levels change if the assumed surface velocities change by a factor of two.
The X-ray flux vs. color plot suggests that our simple analytic scaling treatment successfully reproduces the observed lower limit of X-ray flux found by Schmitt (1997) in the range F0-M0.

- This suggests that the level of heating by magnetic sources in the coronae of these stars is sufficient to account for most if not all of the X-ray flux.
- Stars earlier that type F0 are expected to have rapidly decreasing thicknesses of their outer convective shells. The low observed X-ray flux for B-V < 0.1 are consistent with both magnetic and acoustic heating models.
- We expect our assumptions of a fully ionized gas with high conductivity will break down for cool M dwarf atmospheres.
- We also find good agree with the lower limit of Mg II flux for K dwarfs. The shallow dependence on color leads us to underestimate the flux for earlier spectral types, so another mechanism such as acoustic heating would be required to produce the minimum observed flux. We cannot address the level of acoustic heating with our dynamo model.
References:

Reid, N. I., & Hawley, S. L. 2000, New Light on Dark Stars (New York: Springer)

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