On the Flux of Magnetic Helicity into the Solar Corona

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ABSTRACT

Solar physicists are intently researching the origin of magnetic helicity observed in CMEs in the heliosphere. In particular, helicity estimates for preeruptive active region are much lower than helicity estimates from flux rope models applied to in situ measurements of eruptive magnetic field structures in the heliosphere. While magnetic helicity is often idealized as passing through the solar photosphere into the corona in one of two ways — via emergence of helicity-carrying fields or via twisting and/or shearing motions acting on already-emerged fields that thread the photosphere — I discuss a third mechanism by which helicity may enter the corona: the emergence of a flux system into a preexisting coronal field. To quantify the helicity flux arising purely from this process, I assume the subvolumes containing the emerging flux system and the preexisting flux system are initially helicity free. After emergence, the frozen-in-flux condition can result in a non-potential field topology, with a current sheet on the separatrix surface between the emerged and preexisting flux systems, and, in general, the system has a non-zero relative helicity. I have analyzed a simple model of the final configuration, a bipole with flux $\Phi_B$ whose axis makes an angle $\mu$ with a background field, and find that the relative helicity of the configuration to be on the order of the characteristic helicity in the emerged flux system, $\Phi_B^2$. In this scenario, with helicity stored in the global topology, eruption of the emerged system and entrainment of some overlying, preexisting field can explain the discrepancy between pre- and post- eruption helicities. I also address the contribution of helicity from this process to the solar cycle helicity budget.

Subject headings:

1. Introduction

Coronal mass ejections (CME’s) are among the primary drivers of space weather (Gosling 1993). Consequently, solar physicists are working intently to understand the mechanisms behind
these eruptive phenomena. Low (2002) argues that CME’s serve an essential role in the solar dynamo, by removing magnetic helicity from the solar atmosphere. In volumes like the corona, where the normal magnetic field, $B_n$, does not vanish on the boundary surface — the photosphere — the term magnetic helicity usually refers to the gauge-invariant relative helicity defined by Berger and Field (1984), which Finn and Antonsen (1985) expressed as

$$\mathcal{H} = \int dV (A + A_P) \cdot (B - B_P) ,$$

where $A$ is a vector potential for $B$, $B_P$ is the unique, current-free magnetic field consistent with $B_n$, and $A_P$ is a vector potential for $B_P$. Flux rope models applied to \textit{in situ} magnetic field measurements at 1 AU from the few CME’s that actually form magnetic clouds (MC’s; Li et al. 2003) indeed suggest that each MC might carry away magnetic helicity on the order of $H_{mC} \lesssim 10^4 \Phi_B^2$ from a bipolar active region with flux of $\Phi_B \sim 10^2 B$ in each polarity (Demoulin \textit{et al.} 2002; Green \textit{et al.} 2002; Nindos and Zhang 2002). The precise origin of the helicity that leaves the upper corona in CME’s is unknown. In a statistical study of CME’s and MC’s, Canfield and his collaborators (2003) assert that an order of magnitude more helicity is often present in MC’s than is obvious in their associated pre-eruptive coronal structures, a finding consistent with the results of Demoulin (2002) and Green (2002). Since helicity is well-conserved in the highly conducting coronal plasma (Berger 1984; Taylor 1974), the helicity observed at 1 AU either entered the corona through the photosphere (Berger and Ruzmaikin 2000; DeVore 2000; Chae 2001; Green \textit{et al.} 2002), or was created in the corona simultaneously with helicity of the opposite sign, which remained behind (Pevtsov 2003).

Canfield \textit{et al.} (2003) suggest the helicity present in MC’s results from magnetic reconnection between the erupting field structure and larger-scale, “background” magnetic flux entrained by the eruptive structure during the ejection process. In this scenario, the helicity measured in MC’s is present in the corona prior to eruption, but is not “stored” in either the eruptive structure or the background field; rather, the helicity resides in the global, non-potential topology of the field configuration. The goal of this paper is to estimate the helicity that might be stored in such a system.

Berger and Field (1984) derived an expression for the helicity flux through a closed surface $S$ into a volume $V$,

$$\frac{d\mathcal{H}}{dt} = \oint_S dS [(B \cdot A_P)v_n - (A_P \cdot \mathbf{v}_\perp) B_n] ,$$

in terms of flows parallel and perpendicular to the normal to $S$ ($v_n$ and $\mathbf{v}_\perp$, respectively). Probably from the structure of this expression, helicity fluxes are often idealized as arising
either from emerging helicity-containing fields (the $v_n$ term) or from twisting and/or shearing motions acting on already-emerged fields (the $B_n$ term). The archetype of the former process is the emergence of twisted magnetic fields (Leka et al. 1996), while archetypes of the latter are the rotation of sunspots (Alexander et al. 2002) and shearing along polarity inversion lines (DeVore and Antiochos 2000).

But the emergence of a helicity-free flux system into a corona containing a potential field can lead to a non-zero flux of helicity into the corona, via contributions from both terms in equation (2). ¹ This may be seen from the form of equation (1), as the post-emergence field, $\mathbf{B}$, will, in general, differ from the potential field, $\mathbf{B}_P$, that matches the same boundary condition, $B_n$, such that the integrand can be non-zero.

The flux of helicity into the corona by this mechanism is the focus of this paper. In the next section, I present a simple method to calculate the helicity flux into the corona via the emergence of a potential, bipolar flux system into a corona containing a pre-existing, potential background field. In section 3., I describe the method used to compute the helicity flux in the resulting field configuration, as a function of the angle between the axis of the bipole and the prevailing direction of the background field. In section 4., I do the calculation and find that, for a bipole containing flux $\Phi_B$ in each polarity, the helicity present after emergence is on the order of $\sim \Phi_B^2$. Finally, in section 5., I discuss the implications of this work for our understanding of helicity transport by in the solar atmosphere, especially by CME’s.

2. A Model of Field Evolution

Imagine the solar corona intially contains an essentially dipolar potential field due entirely to isolated, oppositely-signed photospheric fluxes at the polar caps; let us denote this field $\mathbf{B}_P^{(0)}(r, \theta, \phi)$, and the corresponding photospheric flux distribution as $B_n^{(0)}(\theta, \phi)$. The photosphere is assumed to be perfectly conducting.

Now imagine the following “emergence scenario:” a bipolar flux system emerges through a previously flux-free region of the photosphere into the corona, and, further, that the coronal field evolves in response to the change in the boundary flux distribution subject to weak viscous dissipation but no magnetic diffusion. (In the solar case, weak magnetic diffusion

¹Though the helicity density $(\mathbf{A} \cdot \mathbf{B})$ in the emerging flux system is zero, the dot product of the average vector potential and the average magnetic field at the photosphere, in regions where the flux systems meet, does not necessarily vanish.
would be present. Assuming the viscous dissipation timescale is much greater than the emergence timescale, but much less than the magnetic diffusion time scale, would lead to essentially equivalent evolution.)

After a time, field evolution will cease, resulting in a final field \( \mathbf{B}^{(f)}(r, \theta, \phi) \), and final boundary flux distribution \( B_n^{(f)}(\theta, \phi) \). As magnetic reconnection cannot occur without magnetic diffusivity, no field lines link the emerged and pre-existing flux systems. I note that, in general, the potential field \( \mathbf{B}^{(f)}_P(r, \theta, \phi) \) matching \( B_n^{(f)} \) differs from \( \mathbf{B}^{(f)} \).

My goal is to calculate the relative helicity present in \( \mathbf{B}^{(f)} \), assuming the emergent flux system was helicity-free. Evaluation of the integral in equation (1) requires knowledge of four fields: \( \mathbf{B}^{(f)}_P, \mathbf{A}^{(f)}_P, \mathbf{B}^{(f)}_B \), and \( \mathbf{A}^{(f)}_B \). Numerically, at least, \( \mathbf{B}^{(f)}_P \) and \( \mathbf{A}^{(f)}_P \) can be found from any boundary condition, and \( \mathbf{A}^{(f)} \) can be found if \( \mathbf{B}^{(f)} \) is known. In the field configuration that interests me, however, \( \mathbf{B}^{(f)}_P \) is not known, and cannot easily be found: the field is not potential, and must possess high degree of symmetry to be analytically tractable. An MHD code or something like one would be required to solve for the field evolution in the scenario outlined in the previous paragraph; but even with such a tool, modelling the emergence process would be a non-trivial undertaking.

Instead, I adopt a “displacement scenario” to calculate the helicity in \( \mathbf{B}^{(f)} \): starting with an initially potential (helicity-free) state, \( \mathbf{B}^{(i)}_P(r, \theta, \phi) \) that matches a boundary condition \( B_n^{(i)}(\theta, \phi) \) that differs from \( B_n^{(f)} \), I use equation (2) to determine the helicity flux through the photosphere resulting from evolving the photospheric flux distribution to match \( B_n^{(f)} \). A final coronal field, \( \mathbf{B}^{(f)}(r, \theta, \phi) \), would result from this alternative boundary evolution. Assuming, as above, that the coronal field evolves with weak viscosity but without magnetic diffusion, no helicity is dissipated in the coronal volume, and any helicity injected in evolving the photospheric boundary from \( B_n^{(i)} \) to \( B_n^{(f)} \) must be present in \( \mathbf{B}^{(f)} \). I assert that, by making particular choices of \( B_n^{(i)}, B_n^{(f)}, \) and boundary evolution, \( \mathbf{B}^{(f)} = \mathbf{B}^{(f)}_B \) can obtain. In such a case, the helicity present in \( \mathbf{B}^{(f)} \), must be identical to the helicity present in \( \mathbf{B}^{(f)}_B \).

To accomplish this, I assume the pre-existing field arises from two equal but opposite flux elements lying on the ends of a diameter through the Sun, with a “background” flux distribution \( B_n^{(0)} \) given by

\[
B_n^{(0)}(\theta, \phi) = \frac{\Phi_0}{R^2_\odot} \delta(\cos(\theta_0) - \cos(\theta))\delta(\phi) - \frac{\Phi_0}{R^2_\odot} \delta(\cos(\theta_0 + \pi/2) - \cos(\theta))\delta(\pi - \phi) , \tag{3}
\]

and that the emerged bipole’s flux system arises from two equal and opposite fluxes displaced in opposite directions from \( \theta = 0 \) by an angle \( \theta_B \),

\[
B_n^{(B)}(\theta, \phi) = \frac{\Phi_B}{R^2_\odot} \delta(\cos(\theta_B) - \cos(\theta))\delta(\phi - \phi) - \frac{\Phi_B}{R^2_\odot} \delta(\cos(\theta_B) - \cos(\theta))\delta(\phi_B + \pi - \phi) . \tag{4}
\]
In the emergence scenario, going from $B_n^{(0)}$ to $B_n^{(f)}$, $\phi_B$ is kept fixed at $\mu$ while $\theta_B$ increases from zero to a small angle $\theta_{AR}$, a typical angular separation of active region polarities. In the displacement scenario, going from $B_n^{(i)}$ to $B_n^{(f)}$, $\theta_B$ is kept fixed at $\theta_{AR}$, and a neighborhood of the photosphere encompassing $\theta > \theta_{AR}$ (but not any flux from $B_n^{(0)}$) is rotated rigidly about $\theta = 0$, such that $\phi_B$ increases from zero to $\mu$. The boundary condition $B_n^{(i)}$ is shown in the left panel figure 1 (but, for the sake of visualization, point sources are not used), where $\Phi_0/\Phi_B = 1$, $\theta_0 = 75$ deg, $\mu = 90$ deg, and $\theta_{AR} = 5$ deg. The boundary condition $B_n^{(f)}$ is shown in the right panel figure 1 (again, point sources are not used) for similar choices of $\Phi_0/\Phi_B, \theta_0, \mu$ and $\theta_{AR}$. These choices correspond to an active region containing $10^{22}$ Mx of flux in each polarity, emerging at 15 deg N heliographic latitude, with poles separated by $\sim 120 Mm$, with polar cap fields containing $10^{22}$ Mx of flux of each polarity. For these parameters, the bipole in $B_n^{(f)}$ obeys Hale’s Law.

The essential point of this exercise is that the field line topologies in $B_n^{(f)}$ and $\tilde{B}_n^{(f)}$ are the same. In both cases, the bipole’s flux is completely isolated from background field by a separatrix surface: in $B_n^{(f)}$, since reconnection-free emergence was assumed, and in $\tilde{B}_n^{(f)}$, since reconnection-free displacement was assumed, and the bipole’s flux was enclosed within a separatrix in $B_n^{(i)}$, by symmetry (the bipole’s axis pointed in the direction of the background field).

### 3. Helicity Calculations

There are four $\delta$-function sources to consider in evaluating equation (2) in the displacement scenario, two in the background distribution, $B_n^{(0)}$, and two in the bipole distribution, $B_n^{(B)}$. In principle, each source can have a self helicity flux, and each pair of sources can have a mutual helicity flux (Berger 1999).

#### 3.1. Helicity Flux within Bipole System

I begin calculating the helicity flux into the bipole’s flux system. Recall that the displacement is achieved by rigidly rotating a region of the photosphere containing the bipole’s fluxes, and their immediate neighborhood; assume the rotation rate is $-\Omega_B$ (negative with respect to an outward normal, since the motion is clockwise). Berger (1999) has derived a result which is applicable in this case,

$$
\frac{d\mathcal{H}_B}{dt} = -\frac{1}{\pi} [2\Omega_+/(\Phi_+)(\Phi_-) + \omega_+(\Phi_-)^2 + \omega_-(\Phi_+)^2],
$$

(5)
Fig. 1.— Idealized magnetograms of: a) $B_n^{(0)}$; b) $B_n^{(i)}$; and c) $B_n^{(f)}$. In the emergence scenario described in section 2, a bipole emerges in an initially flux-free region of a) to give the boundary flux distribution in c). In the displacement scenario described in section 2, the vertical bipole in panel b) is rotated clockwise through $\mu = 90\,\text{deg}$ until the bipole’s axis is horizontal in panel c). The calculations in this work assume a spherical coordinate system with the north pole at the bipole’s center, marked with a “+,” while a coordinate system with the north pole at the “×” matches the heliographic coordinate system; the heliographic equator is shown as a dashed line. In the calculations of sections 2 and 3, point sources of magnetic flux were used, while sources with finite size are used here for visualization.
where $\omega_\pm$ is the angular rate of rotation of flux $\Phi_\pm$ about its own axis, and $\Omega_{+-}$ is the angular rate of rotation of $\Phi_+$ about $\Phi_-$. (The first term is the mutual helicity flux, and the other two terms are the self helicity fluxes.) Since $\Phi_+ = -\Phi_- = \Phi_B$, and $\Omega_{+-} = \omega_- = \omega_+ = -\Omega_B$ for a rigid rotation, $dH_B/dt = 0$. Hence, there is no flux of helicity into the bipole’s field.

3.2. Self Helicity Flux from Background Sources

As the background fluxes in $B_0^{(0)}$ do not move in the displacement scenario, they do not contribute any self helicity.

3.3. Mutual Helicity Flux Between Bipole and Background

In fact, the only source of helicity flux in the displacement scenario arises from the mutual helicity flux between the bipolar sources and the background sources, in particular, the normal flux from the former, $B_n B$, moving through the vector potential of the latter, $A_P^{(0)}$:

$$
\frac{dH}{dt} = -2 \int_0^\pi d\theta \sin(\theta) \int_0^{2\pi} d\phi \left( A_P^{(0)} \cdot (-\Omega_B \sin(\theta) \hat{\phi}) \right) B_n^{(B)} .
$$

In the heliographic coordinate system $(\theta', \phi')$, with the background fluxes at the north and south poles, the vector potential is circumferential, and given is by

$$
A_P(\theta') = \frac{\Phi_0}{2\pi R \sin(\theta')} \hat{\phi}'.
$$

Transforming this into the frame used in equation (3) gives

$$
A_P(\theta, \phi) = \frac{\Phi_0}{2\pi R} \frac{\hat{r}_0 \times \hat{r}}{|\hat{r}_0 \times \hat{r}|^2} ,
$$

where $\hat{r}_0 = \sin(\theta_0)\hat{x} + \cos(\theta_0)\hat{z}$ points from the origin toward the positive background flux $\Phi_0$, and $\hat{r}$ is the usual radial unit vector in spherical coordinates. We then have

$$
\frac{1}{H_{0B}} dH/d\phi_B = \frac{1}{\pi} \left\{ \frac{\cos(\theta_0) \sin(\theta_AR) - \sin(\theta_0) \cos(\theta_AR) \cos(\phi_B)}{|\hat{r}_0 \times \hat{r}_B^{(+)}|^2} 
- \frac{\cos(\theta_0) \sin(\theta_AR) + \sin(\theta_0) \cos(\theta_AR) \cos(\phi_B)}{|\hat{r}_0 \times \hat{r}_B^{-(-)}|^2} \right\} ,
$$

where $\hat{r}_B^{(\pm)}$ points from the origin toward $\pm \Phi_B$, and where we have used to $\phi_B = \Omega_B t$ to change the helicity flux rate from a function of time to a function of the angular displacement.
of the bipole’s fluxes from their initial position, and have normalized the flux rate by the characteristic helicity of the system, \( \mathcal{H}_{0B} \equiv (\Phi_0 \Phi_B) \). In the top panel of figure 2, we show the helicity flux rate \( d\mathcal{H}/d\phi_B \) as a function of \( \phi_B \), normalized in this way.

To find the total change in helicity \( \Delta \mathcal{H} \), normalized by \( \mathcal{H}_{0B} \) we integrate the expression above over the interval \([0, \mu]\),

\[
\frac{1}{\mathcal{H}_{0B}} \Delta \mathcal{H} = \frac{1}{\pi} \int_0^\mu d\phi_B \frac{d\mathcal{H}}{d\phi_B} .
\] (11)

In the bottom panel of figure 2, we show the normalized, integrated helicity flux as a function of the final rotation angle \( \mu \). This result is essentially that of Berger (1998): (Berger et al. 1998) given two isolated flux tubes of fluxes \( \Phi_A \) and \( \Phi_B \), anchored at a conducting surface, whose axes make an angle \( \mu \), their mutual helicity is \( \mathcal{H}_{AB} = (\mu) \text{mod}(\pi) \Phi_A \Phi_B \).

4. Discussion

Rotating the bipole in the manner described results in a current sheet along the separatrix surface between the bipole and background fields. Such a current layer would also be present if the bipole were to have emerged into a background field. This “topological current” is entirely a result of coronal field’s evolutionary history, and some of this current can be dissipated by magnetic reconnection. In contrast, when a twisted flux rope emerges through the photosphere, it imposes a “boundary current.” Because such currents are driven by the dynamo, which acts as a current driver, reconnection in the coronal field can, at most, temporarily diminish boundary currents (Longcope and Welsch 2000).

While reconnection in this configuration could dissipate some component of this topological current, releasing energy in the process, Berger has shown that dissipation rate of any helicity present in the field would be much lower. If the post reconnection field were not be helicity-free, it would not be potential, so reductive relaxation to the potential state would be impossible. Hence, reconnection might not be able to liberate all of the magnetic free energy (defined to be that above the potential).

In the particular case of \( \mu = \pi \), the potential state can be achieved through reconnection: a current sheet certainly exists between the flux systems, but the field contains no helicity, and a particular sequence of field line reconnections can completely dissipate the topological current. Zhang & Low (2002) have studied similar field arrangements, achieved by flipping field line directions. Doing so results in a field that is far from potential, but for which the relative helicity vanishes: their assumed geometry is axisymmetric, with toroidal vector potentials and poloidal magnetic fields, such that the dot product in equation (1) vanishes.
Fig. 2.— In the top panel, we show the instantaneous rate of helicity flux from rotating the bipole’s fluxes, as a function rotation angle $\phi_B$, normalized by the characteristic helicity of the system, $H_{0B}$. In the bottom panel, we show the total helicity flux from rotating the bipole’s fluxes through an angle $\mu$, normalized by the characteristic helicity of the system, $H_{0B}$. In both panels, the thin solid line is the helicity flux from the bipole’s negative magnetic flux, the dashed line is the helicity flux from the bipole’s positive magnetic flux, and the thick solid line is the sum of the two.
While the coronal field can contain free energy without helicity, it cannot contain helicity without energy above the potential state. The flux of helicity into the corona, then, can inform us of the extent to which the coronal field deviates from the potential state. Since the coronal field evolves toward the potential state, at some point, ejecting helicity present in the field configuration might become energetically favorable.

Whether the emergence scenario or the displacement scenario leads to the final state described above, the configuration contains a helicity on the order of the characteristic helicity of the two flux systems, \( \mathcal{H}_B = \Phi_0 \Phi_B \). Assuming \( \Phi_B \sim \Phi_0 \) — reasonable for the Sun — the field contains a helicity on the order of \( \Phi_B^2 \). If the emergent flux were to erupt, and entrain the overlying field in the process, some component of this helicity could contribute to the helicity measured in the solar wind at 1 AU.

Besides the eruptive transport of helicity from the low corona, this helicity flux process is at work with each emerging active region over a solar cycle. As above, assuming a helicity flux of \( \Phi_B^2 \sim 10^{44} \text{Mx}^2 \) per active region, and \( N \sim 1500 \) emergences per hemisphere over an 11-year cycle, the whole-cycle helicity flux via the emergence scenario ought to be of order \( \mathcal{H}_{\text{CYC}} \sim 10^{47} \text{Mx}^2 \) per hemisphere. As this process couples emerging active regions to the large-scale background field, the handedness of the helicity depends upon that field. If the flux of helicity flips sign across the equator, much of this helicity could cancel across the equator by reconnection in transequatorial interconnecting loops (TILs).

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