Three-dimensional numerical simulation of MHD waves observed by the Extreme Ultraviolet Imaging Telescope

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Abstract. We investigate the global large amplitude waves propagating across the solar disk as observed by the SOHO/Extreme Ultraviolet Imaging Telescope (EIT). These waves appear to be similar to those observed in Hα in the chromosphere and which are known as “Moreton waves,” associated with large solar flares [Moreton, 1960, 1964]. Uchida [1968] interpreted these Moreton waves as the propagation of a hydromagnetic disturbance in the corona with its frontface intersecting the chromosphere to produce the Moreton wave as observed in movie sequences of Hα images. To search for an understanding of the physical characteristics of these newly observed EIT waves, we constructed a three-dimensional, time-dependent, numerical magnetohydrodynamic (MHD) model. Measured global magnetic fields, obtained from the Wilcox Solar Observatory (WSO) at Stanford University, are used as the initial magnetic field to investigate hydromagnetic wave propagation in a three-dimensional spherical geometry. Using magnetohydrodynamic wave theory together with simulation, we are able to identify these observed EIT waves as fast mode MHD waves dominated by the acoustic mode, called magnetosonic waves. The results to be presented include the following: (1) comparison of observed and simulated morphology projected on the disk and the distance-time curves on the solar disk; (2) three-dimensional evolution of the disturbed magnetic field lines at various viewing angles; (3) evolution of the plasma density profile at a specific location as a function of latitude; and (4) computed Friedrich’s diagrams to identify the MHD wave characteristics.

1. Introduction

Observations of wave phenomena propagating across the solar disk were made by a number of investigators in the early 1960s [Athay and Moreton, 1961; Moreton, 1960, 1964], and referred to as “Moreton waves.” This phenomenon was seen in movie sequences of the Hα emission, which showed successive semicircular “fronts” expanding from a flare with speeds ranging from a few hundred to well over 10³ km s⁻¹. Higher speeds have been observed. An unusually extreme example of Moreton wave’s acceleration from 2500 to 4000 km s⁻¹ for the X12/3B flare of June 4, 1991, has been reported by Sakurai et al. [1995]. Transient coronal EUV emission has also been observed prior to and during the initial stage of a solar flare [Neupert, 1989]. Neupert’s data showed the presence of a wave-like disturbance traveling with a speed of ~760 km s⁻¹, expanding from a site along the neutral line. He suggested that it may have been the location of a chromospheric mass ejection, but he did not present the global morphology of this observed wave.

The SOHO/Extreme Ultraviolet Imaging Telescope (EIT) images the solar disk and inner corona [Delaboudinière et al., 1995]. On May 12, 1997, an Earth-directed coronal mass ejection (CME) was observed by the SOHO/EIT and reported by Thompson et al. [1998]. The CME, originating north of the central solar meridian, was later observed by the Large Angle Spectrometric Coronagraph (LASCO) as a “halo” CME (i.e., a bright expanding ring centered about the occulting disk as reported by Plunkett et al. [1998]). Thompson et al. [1998] reported the details of the EIT observations of this CME event; we will not repeat them here. The focus of our study is to understand the physics of the observed, almost circularly shaped, wave propagating across the solar disk with a measured average speed of ~250 km s⁻¹. This particular speed is much slower than the speed of a Moreton wave in the chromosphere-corona transition region.

These observed waves (i.e., Moreton waves and presumably, Neupert’s transient coronal emission waves) have been attributed to MHD fast mode waves generated in the impulsive phase of the flare [Meyer, 1968; Uchida, 1968, 1974; Uchida et al., 1973]. This seems to be a reasonable interpretation; however, the recently observed EIT waves have distinct characteristics which differ from previously observed Moreton waves [Moreton, 1960] and transient coronal emissions [Neupert, 1989]. These differences are (1) the speed of the EIT wave is much less than the Moreton wave with propagation speeds on
the order of the local Alfvénic speed; and (2) the morphology of the EIT wave propagation can be almost circular across the solar disk, while Moreton waves were typically confined to an arc rarely exceeding 120°.

In this paper, we present a three-dimensional, time-dependent magnetohydrodynamic model using observed magnetic fields and assumed solar plasma properties (no observations are available) to investigate the propagating, finite amplitude, global MHD waves on the solar disk. We pursue the approach in order to understand the observed EIT waves and their relationship with Moreton waves. The MHD model and initial boundary conditions will be described in section 2. Numerical results and analysis will be included in section 3. The concluding remarks are given in section 4.

2. Model Description

A three-dimensional, time-dependent ideal (nondissipative) numerical MHD model is used. For simplicity, we approximate the energy equation with an adiabatic energy process with the polytropic index ($\gamma$) being 1.67. The other equations include mass conservation, momentum conservation, and magnetic induction to take into account the nonlinear interaction between plasma flow and magnetic fields. The mathematical model is presented in a matrix form in spherical coordinates for convenience of development of the numerical code, namely,

$$\frac{\partial \mathbf{u}}{\partial t} + A_r \frac{\partial \mathbf{u}}{\partial r} + A_s \frac{1}{r} \frac{\partial \mathbf{u}}{\partial \theta} + A_e \frac{1}{r \sin \theta} \frac{\partial \mathbf{u}}{\partial \phi} + \mathbf{F} = 0,$$  (1)
Figure 3. The spatial distribution of the pressure pulse. This pulse linearly increases to a maximum of a factor of 2 times the background pressure at 398 s. The pulse remains constant until 955 s, then decreases to the initial pressure at 1910 s.

with

$$
\begin{bmatrix}
\rho \\
v_x \\
v_y \\
v_z \\
B_x \\
B_y \\
B_z \\
p
\end{bmatrix}
= A_u
$$

$$
A_u =
\begin{bmatrix}
v_x & p & 0 & 0 & 0 & 0 & 0 & 0 \\
v_y & 0 & v_x & 0 & 0 & 0 & 0 & 0 \\
v_z & 0 & 0 & v_y & 0 & 0 & 0 & 0 \\
B_x & 0 & 0 & 0 & v_z & 0 & 0 & 0 \\
B_y & 0 & 0 & 0 & 0 & v_x & 0 & 0 \\
B_z & 0 & 0 & 0 & 0 & 0 & v_y & 0 \\
\gamma p & 0 & 0 & 0 & 0 & 0 & 0 & v_z
\end{bmatrix}
$$

$$
A_v =
\begin{bmatrix}
v_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_y & 0 & v_x & 0 & 0 & 0 & 0 & 0 \\
v_z & 0 & 0 & v_y & 0 & 0 & 0 & 0 \\
B_x & 0 & 0 & 0 & v_z & 0 & 0 & 0 \\
B_y & 0 & 0 & 0 & 0 & v_x & 0 & 0 \\
B_z & 0 & 0 & 0 & 0 & 0 & v_y & 0 \\
\gamma p & 0 & 0 & 0 & 0 & 0 & 0 & v_z
\end{bmatrix}
$$

$$
A_u =
\begin{bmatrix}
v_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_y & 0 & 0 & 0 & 0 & -B_x/p & 0 & 0 \\
v_z & 0 & 0 & 0 & 0 & 0 & -B_y/p & 0 \\
B_x & 0 & 0 & 0 & 0 & 0 & 0 & \beta/2p \\
B_y & 0 & 0 & -B_x/p & 0 & 0 & 0 & 0 \\
B_z & 0 & -B_y/p & 0 & 0 & 0 & 0 & 0 \\
\gamma p & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
A_v =
\begin{bmatrix}
v_x & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
v_z & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
B_x & 0 & 0 & 0 & 0 & 0 & 0 & \beta/2p \\
B_y & 0 & 0 & -B_x/p & 0 & 0 & 0 & 0 \\
B_z & 0 & -B_y/p & 0 & 0 & 0 & 0 & 0 \\
\gamma p & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$
\[ \hat{F} = \frac{1}{r} \begin{bmatrix} \rho(2v_r + v_\theta \cos \theta) \\ -(v^2_r + v^2_\theta) + (B^2_r + B^2_\theta)/\rho + \alpha/r \\ v_r v_\theta - v^2_\theta \cot \theta - (B_r B_\theta - B^2_\phi \cot \theta)/\rho \\ v_r(v_r + v_\phi \cot \theta) - B_r(B_r + B_\phi \cot \theta)/\rho \\ B_r(2v_r + v_\phi \cos \theta) \\ v_r B_\phi + v_\phi(B_r + B_\phi \cot \theta) \\ v_r B_\phi + v_\phi(B_r + B_\phi \cot \theta) \\ \gamma p(2v_r + v_\phi \cos \theta) \end{bmatrix} \]

where \( t, r, \rho, \vec{u}(v_r, v_\theta, v_\phi), \vec{B}(B_r, B_\theta, B_\phi) \), \( p \) are the time, radius, density, velocity vector, magnetic field vector and thermal pressure, normalized by \( \tau_\lambda, R_s, \rho_0, V_A, B_0, p_0 \). Additional symbols are \( \theta \) (latitude), \( \varphi \) (longitude), \( \beta = 8\pi p_0/B_0^2, V_A = B_0/\sqrt{4\pi \rho_0}, \alpha = GM/R_s V_A^2, \tau_\lambda = R_s/V_A \), where \( R_s \) is the solar radius, \( M_s \) the solar mass, \( G \) the gravitational constant.

The numerical scheme used to develop the MHD model is the fractional step method [Yanenko, 1971]. This numerical scheme consists of a preliminary step and correction steps. The implicit scheme [Roache, 1999] is used for maximum stability at the preliminary step with respect to each coordinate \( (r, \theta, \varphi) \). The explicit scheme [Roache, 1999] is used for the correction step for accuracy. The overall accuracy of this numerical

Figure 4. Comparison between the SOHO/EIT 195 Å running-difference images of the large scale wave transient and numerically simulated density-enhancement images of a large-scale wave transient on the solar disk. The simulated density enhancements compare very well with the EIT intensity enhancement at each of the four times shown here.

Figure 5. The measured and numerically simulated distance-time curves of the wavefronts shown in Figure 4 in the north, south, southwest, and southeast directions. The numerically simulated wave fronts are identified by the 10% level of the density enhancement.
scheme is tested by using an analytical solution given by Low [1984], which shows that the difference between the numerical and analytical solution is within 5% over the time period of the model calculation.

The initial state of this model is a magnetohydrostatic solar atmosphere; thus, we can choose the current-free (i.e., potential) magnetic field topology as the initial state. In this study the measured photospheric global-scale, line-of-sight magnetic field map (Carrington rotation 1922) of May 12, 1997, shown in Figure 1 from the Stanford Wilcox Solar Observatory is used together with the source-surface model [Hoeksema and Scherrer, 1985; Hoeksema, 1989, 1991]. Because of the low spatial resolution of the magnetograph at WSO, the photospheric field is concentrated into subarc second bundles of strong field that could not be measured. That restriction gives the measured photospheric weak field on the order of a gauss as shown in Figure 1. Physically, this representation of global field distribution is a reasonable approximation, because at small distances above the photosphere, the field is no longer concentrated into small bundles of radial field. The magnetic flux rapidly spreads into a more uniform distribution. This interpretation is supported by the change in field strength across the disk of various features measured using spectral lines which form at different heights [Hoeksema, 1984]. Because of the lack of number density ($n$) and temperature ($T$) measurements just above the solar surface at the coronal base, we simply choose typical values of corona given by Feldman et al. [1999]: $n_o = 1.2 \times 10^8$ cm$^{-3}$ and $T_o = 1.6 \times 10^6$ K. These values are uniformly distributed on the solar disk at this level. It should be understood that this choice is not realistic because the density varies from pole to equator on the solar surface; however, the present interest is to examine the propagation of MHD waves in a region away from the poles. This choice should be appropriate. We obtained, Figure 2, the plasma beta (i.e., the ratio of plasma pressure to the magnetic pressure $16\pi nkT/B^2$) using the measured magnetic fields from Figure 1 together with the assumed plasma density and temperature. The vertical row of numbered dots at Carrington longitude ~135° will be discussed later.

The lower and upper boundary conditions in the radial direction are adopted from the time-dependent characteristic boundary conditions. This requirement means that the values for all the physical quantities are updated for each time-step according to the compatibility equations derived from the governing equations [Hu and Wu, 1984; Wu and Wang, 1987]. The nonreflecting conditions are used according to the direction of characteristics in the radial direction. Periodic boundaries are used for the latitudinal and longitudinal directions. Specific mathematical expressions used for the radial boundary conditions are included in Appendix A.

3. Numerical Results and Analyses

From the observations, EIT images show an almost circular expanding wave front, emanating from a central point near, or at, NOAA active region 8038 at N23W07 from 0450 to 0700...
UT, May 12, 1997. To simulate this wave front, we introduce a thermal pressure pulse of a factor 2 higher than the background pressure. The temporal and spatial shapes of the pulse are shown in Figure 3. The total energy input corresponding to this pulse is $5 \times 10^{28}$ ergs. Physically, this pressure pulse may be interpreted as representing active region heating which results from complex processes that are not considered in this paper. The EIT images (top) and model-simulated images showing computed density enhancements (bottom) are shown in Figure 4. By comparing the simulation results with observations, it is clearly indicated that the simulated and observed wave fronts are matched well as shown in Figure 4. The simulated wave is the representation of density enhancements due to the given disturbance at four representative times. Further, we have plotted the measured and simulated distance and time curves in various directions (i.e., south, north, southeast, and southwest) in Figure 5. These kinematical characteristics also exhibit remarkable agreement.

By noting the results shown in Figures 4 and 5, there is a feature which needs explanation, that is: both figures indicate that the density enhancement does not move any further along the northern direction after it reaches a location $600$ Mm from the initiation site (see Figure 5) where it remains. This can be understood by examining the magnetic field measure-
Figure 8. The computer evolution of the density enhancement, \((\rho - \rho_0)/\rho_0\) viewed at central meridian (CM), 45°, and 90° away from central meridian (where the explosion occurred) at 1910 and 2865 s, respectively.

Figure 9. MHD fast wave Mach number at 955, 1432, 1910, and 2865 s after introduction of the perturbation with their maximum values being 0.58, 0.61, 0.53, and 0.48, respectively.
ments shown in Figure 1, where it can be seen that this position corresponds to the southern boundary of the north polar coronal hole. The physical explanation of this “anomaly” is as follows. When these large-amplitude nonlinear MHD waves hit the southern boundary of the north polar coronal hole, any associated mass motion across the magnetic field lines because of the frozen-in condition is impeded. This means that the field and plasma move together. By looking at the plasma beta ($\beta$) distribution shown in Figure 2 it is clear that the $\beta$ is larger in the neighborhood of the disturbance and, close to the coronal hole boundary, the $\beta$ becomes much smaller. In such a situation, the large beta ($\beta$) plasma and field medium is easily moved around by the plasma flow; whereas, in the small beta plasma and field medium, the motion is much more rigid and hence constrained. Therefore, when the disturbances reach the southern boundary of the north polar coronal hole the mass motion basically stops. That is why the plasma and field pile up against the coronal hole boundary. It can be seen from Figure 4 that the northern MHD wave front brightens up as a result of the increased plasma density. However, fast mode MHD waves leak out into the coronal hole as shown in Figure 6, which shows the relative density enhancement $(\rho - \rho_o)/\rho_o$ as a function of latitudinal angle on the solar surface, oriented at the center of the meridian of the observed location of the active region at various times. The two dashed lines represent the northern and southern polar coronal hole boundaries. The measurements of Figure 1 show that the distances between the center of the active region (NOAA 8038; 23°N of disk center) and the northern and southern boundaries of the coronal hole are 650 and 1200 Mm, respectively.

We now understand that as the wave front propagates in the northern direction, it asymptotically approaches a distance of $\sim$700 Mm from the initiation site as shown in Figure 5. On the other hand, the wave continues to propagate along the south, southeast and southwest directions because these waves have not reached the boundary of the southern polar coronal hole. Figure 6 shows the evolution of relative density $(\rho - \rho_o)/\rho_o$ after introduction of the initial pressure pulse as shown in Figure 3. This pressure pulse reaches its peak at 955 s and gradually declines to zero at 1910 s. By looking at these results, we notice that the plasma has piled-up at the southern boundary of the northern polar coronal hole as indicated by the results shown in Figures 4 and 5. It is also noted that there is no density enhancement at the northern edge of the southern polar coronal hole because the wave, as yet, has not reached the southern polar coronal hole boundary. It is worth noting in Figure 6 that the compression and rarefaction of those waves are still enhanced after the initial pulse was terminated. Because of the frozen-in condition, the plasma moving with the magnetic fields hits the coronal hole boundary (very small plasma beta) and bounces back (i.e., reflected) to cause these compressions and rarefactions. We have checked the total mass which is conserved.

At this point we have identified the observed EIT waves as MHD waves and explained the northward and southward propagation of the density enhancements as they approach the
coronal hole boundaries. However, we have not, as yet, mentioned the modes of the MHD waves. It is well known from MHD wave theory \cite{Jeffrey and Taniuti, 1964} that when a disturbance appears in a magnetized plasma, three modes of MHD waves, (i.e., Alfven, fast and slow modes) will always be generated. In this study, we focus our attention on the two compressive modes which are characterized by the fast and slow modes and their characteristic speeds as follows:

\[ C_f, C_s \approx \frac{a^2}{H_1 (1 - \frac{\Theta}{100})}, \frac{s^2}{H_2 (1 - \frac{\Theta}{100})} \]

where \( a \) and \( b \) are the sonic and Alfven speeds, respectively, and \( \Theta \) is the angle between the magnetic field and propagating wave (\( \hat{k} \)).

In order to understand further these MHD wave characteristics in this particular case, we constructed Friedrich’s diagrams \cite{Bazer and Fleischman, 1954; Friedrichs, 1954; Friedrich and Kranzer, 1958} at the locations marked by the numbered dots on Figure 1, as shown in Figure 7. The Friedrich’s diagrams are constructed by using the specific physical parameters (i.e., field strength and direction, density and temperature) at each of the numbered positions (1–9) when the wave is generated by a given perturbation at these specific positions. The Friedrich’s diagram, representing the phase polars of hydro-magnetic waves, explains the MHD waves’ generation and propagation. Figure 7 represents the magnitude of the 4 characteristic speeds that propagate in a specific direction referring to the initial magnetic field direction as shown in each panel. The direction of the initial magnetic field is determined by the WSO measurements together with the potential field model. By looking at these results, we recognize that the fast-mode MHD wave, represented by the dashed line, propagates almost in a circular shape which implies isotropic wave propagation. It is apparent that this fast-mode wave resembles the observed EIT waves. This is obvious from Figure 2 because the background is a high plasma-beta medium. When the MHD wave propagates in a high plasma beta medium, it degenerates to a pressure wave as if it were to propagate in a nonmagnetized medium. Specifically, let us examine the Friedrich’s diagrams at location 4 where the disturbance occurs as observed. The characteristic speed of the fast-mode MHD wave at this position is almost equal to the acoustic wave speed. This implies that this particular observed MHD fast-mode wave is dominated by the acoustic mode that is the magnetosonic wave. This result is consistent with the plasma-beta (i.e., \( \beta \) much larger than unity) that was deduced from the measured magnetic field strength shown in Figure 2.

Figure 11. Normalized tangential magnetic field \( (B_t) \), normalized density \( \rho \), pressure \( p \), and entropy \( s \), in the radial direction at specific locations of latitude (\( \theta \)) and Carrington longitude (\( \varphi \)) referring to the region of the inner box shown in Figure 10 and \( t = 1433 \) sec. Plus signs indicate the location where the MHD fast shock-jump conditions are satisfied. Solid lines in the first (top) two rows represent the \( B_t \). Solid/dotted/dashed lines in the second two rows are density \( \rho \), pressure \( p \), and entropy \( s \), respectively. When there is no plus sign, this implies that the shock jump condition is not satisfied. As shown in Figure 10, it corresponds to where the flow speed does not cross over the MHD fast wave speed.
Let us examine this wave further in three dimensions. Figure 8 shows the density enhancement (i.e., wave front) at two times with various viewing angles. When it is viewed from central meridian on the solar disk (leftmost panel), the wave front resembles the wave front seen by EIT. When it is viewed at 45° and 90° from the central meridian, the three-dimensional feature of these MHD waves are clearly indicated. It also could be seen that the edge of the wave front intersects the layer of solar atmosphere on the disk to mark the positions as the wave front seen by EIT. By looking at Figure 8, it is apparent that this magnetosonic fast wave has a three-dimensional shell topology. This phenomenon was also suggested by Uchida [1968] and modeled by using a linear ray-tracing analysis [Uchida et al., 1973] to interpret the generation and propagation of the Moreton wave. That is, the edge or “skirt” of a three-dimensional shell (wave) intersecting a certain layer of the solar atmosphere (i.e., photosphere, chromosphere, and corona) generates the waves at that layer of the atmosphere.

Up-to-now, we have identified this observed EIT wave (May 12, 1997) on the solar disk to be a magnetosonic wave. It is interesting to investigate further whether this wave could be a MHD fast shock. To examine this aspect, we need to examine the ratio of the plasma mass flow speed to the local MHD fast-mode characteristic speed. To obtain this ratio, we need to know the plasma mass flow speed and local MHD fast-mode characteristic speed on the solar disk. According to nonlinear wave theory [Jeffrey and Taniuti, 1964] the wave propagation speed is the plasma mass flow speed plus the local characteristic wave speed. The local characteristic speed is the speed of the MHD fast-mode wave at latitude (θ) and Carrington longitude (φ)-directions on the solar disk which can be computed from (7) by using the local field and plasma parameters on the solar surface. Then, we take the ratio of plasma mass flow speed to the local MHD fast-mode characteristic speed; if this value is larger than unity, a MHD fast shock will appear. This ratio could be defined as the MHD fast wave Mach number, and its results (i.e. contours of the MHD fast wave Mach number) are given in Figure 9. Specifically, the results presented here are the maximum possible MHD fast wave Mach number for different times which are computed by using plasma mass flow speeds (vφ and vθ) and the minimum fast speed on the solar surface. By examining Figure 9, the highest value of the contours are 0.58, 0.61, 0.53, and 0.48, at these times, respectively; thus these observed EIT waves are not MHD fast shocks on the solar disk.

However, it is interesting to know whether there is a possibility that this MHD wave could develop to a MHD fast shock along the radial direction. We have plotted the radial MHD fast speed (Cf,r) and radial flow speed along the radial direction at specific locations (i.e., at a specific latitude (θ) and longitude (φ) on the solar disk) in Figure 10 at t = 1433 s. It should be noted that there is a region (i.e., 13.5° < θ < 32°; 122° < φ < 140°) as marked by the box (dashed-dotted lines) shown in Figure 10, where the radial flow speed would exceed...
the local $C_{fr}$ in some locations, and thus a MHD fast shock could develop. It is interesting to note that the region where the shock may develop is directly above the initiation site at latitude $\theta = 23^\circ$N and the longitude, $\varphi = 135^\circ$. To examine further the existence of MHD fast-mode shocks in this particular region, we test the magnetic field and plasma properties (i.e., density, pressure, and entropy) to see whether they satisfy the shock-jump conditions given in chapter 6, p. 219, and Appendix D of Jeffrey and Taniuti [1964]. These results are shown in Figure 11 for $t = 1433$ sec. The cross marks indicate where the shock-jump conditions are satisfied. By looking at these results, we recognize that the MHD fast-shock jump conditions are satisfied in specific locations in this region. This may be attributed to the local plasma conditions because the local values of plasma beta have deterministic effects on the shock formation [Whang, 1987]. In two locations (i.e., $\theta = 13.5^\circ$, $\varphi = 131.5^\circ$ and $\theta = 22.5^\circ$, $\varphi = 122.5^\circ$), we find the existence of forward MHD fast-shock pairs. When these MHD fast shocks propagate outward beyond our computational domain, it is not clear whether they would survive the low plasma-beta environment [Whang, 1987]. In short, we realize this observed EIT wave has not developed to a MHD fast shock on the solar disk (i.e., at the layer of lower corona), but a weak fast MHD shock does appear when it propagates outward at $-1.6$ solar radii in a limited region directly above this disturbance region. This is obvious, when a point disturbance propagates outward spherically, it will be attenuated away from the disturbance center line.

Finally, we show the disturbed magnetic field lines viewed from central meridian at the various times depicted in Figure 12. The small arrow indicates the location of the disturbance. It shows that the initial potential field used for this simulation has been significantly affected and is now highly nonpotential. However, it is difficult to realize the relationship between the density enhancements and these disturbed magnetic field lines. However, the relationships could be recognized by examining the disturbed field lines away from central meridian. Figure 13 shows the disturbed field lines at various viewing angles (left to right: central meridian, $45^\circ$ and $90^\circ$ from central meridian), respectively, for two different times (i.e., 1910 and 2856 s after introduction of the disturbance). It is easily recognized that these magnetic field lines correspond to the relative density plots shown in Figure 8 where magnetic field lines formed the loops having the character of density enhancements. It is interesting to note that the field lines, viewed at the limb, show rising coronal loops similar to those shown in Figure 8, and which have been observed in some EIT limb events (B. J. Thompson, personal communication, 2000). Another observational feature that could be explained by this simulation is the suggestion of Thompson et al. [1999] that the EIT waves could be generated by the expanding field lines in the solar atmosphere. This feature can be seen simultaneously from the disturbed magnetic field (Figures 12 and 13) and density enhancement (Figure 8). That is, after introduction of the pressure disturbance, the pressure force acts on the fields, then, as a result of the large plasma beta, the magnetic field is not stiff and is easily moved around to generate detection of the EIT waves.

![Figure 13. The computed magnetic field topology corresponding to Figure 8.](image-url)
4. Concluding Remarks

Using a three-dimensional, time-dependent ideal MHD model, the observed EIT waves (May 12, 1997) were, for the first time, identified as MHD-magnetosonic waves, i.e., MHD fast mode waves dominated by the acoustic mode, as shown by the Friedrich’s diagram (Figure 7). The preexistent state of this three-dimensional MHD model is in hydrostatic equilibrium with measured global line-of-sight magnetic fields together with a source-surface model given by the Stanford Wilcox Solar Observatory [Hoeckema and Scherrer, 1985]. Recently Wang [2000] presented a study for these observed EIT waves by using a linear analysis of the ray tracing method. He suggested that the fast-mode MHD wave hypothesis could account for some properties of EIT bright fronts. He has presented an excellent argument to examine the low propagation speed of EIT waves. In our study the low propagation speed of observed EIT was achieved by using WSO global measurements which only accommodate weak fields that lead to a propagation speed of $\sim 260$ km s$^{-1}$. We have run several simulations by simply increasing the measured field by a factor of 2 and 4, respectively to test the plasma beta effect on the wave propagation speed. These tests show that the propagation speed is much faster than the observed EIT wave speed. The simulation results could not match the observation. That is another reason we believe this observed EIT wave is generated in a large plasma beta environment and identified as a magnetosonic wave.

The present MHD numerical simulations reproduced some of the observed features of the May 12, 1997, event observed by EIT. From the observations, the measured propagation speed of this EIT wave is far less than the speed of Moreton waves as reported by Moreton [1961] and Neupert [1989]. We would like to offer the following explanation for this fact.

On the basis of MHD theory [Jeffrey and Taniuti, 1964], the propagation speeds of MHD waves depend on the values of plasma beta. The Moreton wave is measured in a flaring active region environment and is directly related to the magnetic field strength of the active region, where the plasma beta is small. Hence the measured speed of a Moreton wave is of the order of the Alfvén speed ($\sim 10^3$ km/s). On the other hand, the EIT wave propagates in a global-scale environment in which the measured average global-scale magnetic field only accounts for weak fields. As we have shown in Figure 2, the plasma beta is much larger than unity. This leads to an acoustic speed of a factor 2 larger than the Alfvén speed in this case. Therefore we suggest that the earlier observed Moreton wave and Neupert’s coronal EUV transient are MHD fast waves propagating in a local medium directly related to the active region field strength, i.e., in a small plasma-beta medium. On the other hand, the newly observed EIT wave [Thompson et al., 1998] is a MHD fast wave propagating on a global scale in a large plasma-beta medium which is dominated by the acoustic mode and named “magnetosonic wave.” From our analysis this observed EIT wave has not developed into a MHD fast shock on the disk, but there is a small region along the radial direction at $\sim 1.6$ solar radii (Figure 11) where a forward MHD fast shock exists. We have also revealed the existence of forward fast MHD shock pairs in the radial direction. It should be noted that we have not discovered slow-mode MHD waves in the simulation. This may be explained by using the Friedrich’s wave diagrams shown in Figure 7 where the slow-mode MHD waves only appear in a very limited (or restricted) situation.

As a final remark, this three-dimensional magnetohydrodynamic simulation describes an outward bulk motion of plasmas and magnetic fields and an outwardly propagating density that mimics the observed EIT wave. It may be considered as the early stage of a coronal mass ejection.

Appendix A: Boundary Conditions

The boundary conditions used for this simulation are the time-dependent characteristic boundary conditions which are based on previous work [Hu and Wu, 1984; Nakagawa et al., 1987; Wu and Wang, 1987]. However, the numerical procedures to implement these conditions are different than the previous work; thus we briefly summarize below.

Using the governing equations (1)–(6) in the text, we seek the characteristics along the projected normal in the $r$–$t$ plane. Let us denote the left eigenvectors of $A_i$ by $L_i$, thus

$$L_i A_i = \lambda L_i, \quad i = 1, 2, \ldots, 8,$$

(A1)

where the eigenvalues $\lambda_i$ are

$$\lambda_i = \nu_i, \quad \nu_i \pm c_s, \quad \nu_i \pm b_n, \quad \nu_i \pm c_f,$$

(A2)

with

$$\nu_i = \nu, \quad B_n = B_n, \quad B_2 = B_2, \quad B_3 = B_3, \quad b_n = b_n^2, \quad c_s^2 = \frac{1}{2} \left[ a^2 + b^2 \pm \sqrt{(a^2 + b^2)^2 - 4a^2b_n^2} \right].$$

(A3)

$$a^2 = \frac{\beta \gamma \rho \nu}{2}, \quad b^2 = \frac{B_n^2 + B_2^2 + B_3^2}{\rho}.$$  

(A4)

The projected, normal characteristics are described by the equations

$$\frac{dr}{dt} = \nu, \quad \nu \pm c_s, \quad \nu \pm b_n, \quad \nu \pm c_f.$$  

(A5)

The corresponding left eigenvectors are

$$\lambda = \nu, \quad L_1 = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad -1],$$  

(A6)

$$L_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix},$$  

$$L_3 = \begin{bmatrix} 0 & c_s(b_n^2 - c_f^2) & B_2B_s c_s & B_nB_s c_s & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \end{bmatrix},$$  

(A7)

$$\lambda = \nu - c_n, \quad L_4 = \begin{bmatrix} 0 & c_n(b_n^2 - c_f^2) & B_2B_s c_n & B_nB_s c_n & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \end{bmatrix},$$  

(A8)

$$\lambda = \nu - b_n, \quad L_5 = \begin{bmatrix} 0 & 0 & B_3 \text{sign}(B_n) \\ 0 & 0 & -B_3 \text{sign}(B_n) \\ 0 & 0 & B_3 \text{sign}(B_n) \\ 0 & 0 & -B_3 \text{sign}(B_n) \end{bmatrix},$$  

(A9)

$$\lambda = \nu + b_n, \quad L_6 = \begin{bmatrix} 0 & 0 & B_3 \text{sign}(B_n) \\ 0 & 0 & -B_3 \text{sign}(B_n) \\ 0 & 0 & B_3 \text{sign}(B_n) \\ 0 & 0 & -B_3 \text{sign}(B_n) \end{bmatrix},$$  

(A10)

$$\lambda = \nu - c_f, \quad L_7 = \begin{bmatrix} 0 & c_f(b_n^2 - c_f^2) & B_2B_s c_f & B_nB_s c_f & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \end{bmatrix},$$  

(A11)

$$\lambda = \nu - c_f, \quad L_7 = \begin{bmatrix} 0 & c_f(b_n^2 - c_f^2) & B_2B_s c_f & B_nB_s c_f & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & -B_2^2 & -B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & B_3^2 & B_n^2 & 0 \\ 0 & B_2^2 & -B_3^2 & B_n^2 & 0 \end{bmatrix},$$  

(A12)
$$\lambda = v_n + c_f, \quad L_8 \left[ 0 - c_f (b^2_n - c_f) \right] \frac{B_x B x f}{\rho} - \frac{B_y B y f}{\rho} 0 \frac{b^2_n - c_f}{2} \right].$$ (A13)

Let

$$Q = -A_r \frac{\partial u}{\partial r} - A_{\theta} \frac{1}{r} \frac{\partial u}{\partial \theta} - A_{\phi} \frac{1}{r \sin \theta} \frac{\partial u}{\partial \phi} - F.$$ (A14)

The corresponding characteristic equations along these projected normal characteristics are listed with their respective projected normal characteristics.

$$\frac{dr}{dt} = v_n, \quad L_1 \frac{\partial u}{\partial t} = L_3 Q,$$ (A15)

$$\frac{dr}{dt} = v_n - c_f, \quad L_1 \frac{\partial u}{\partial t} = L_3 Q,$$ (A16)

$$\frac{dr}{dt} = v_n + c_f, \quad L_4 \frac{\partial u}{\partial t} = L_3 Q,$$ (A17)

$$\frac{dr}{dt} = v_n - b_n, \quad L_4 \frac{\partial u}{\partial t} = L_3 Q,$$ (A18)

$$\frac{dr}{dt} = v_n + b_n, \quad L_5 \frac{\partial u}{\partial t} = L_3 Q,$$ (A19)

$$\frac{dr}{dt} = v_n - c_f, \quad L_1 \frac{\partial u}{\partial t} = L_3 Q,$$ (A20)

$$\frac{dr}{dt} = v_n + c_f, \quad L_2 \frac{\partial u}{\partial t} = L_3 Q,$$ (A21)

Nonreflecting boundary conditions are stated as

$$L_1 \frac{\partial u}{\partial t} = 0,$$ (A23)

$$L_2 \frac{\partial u}{\partial t} = 0,$$ (A24)

$$L_3 \frac{\partial u}{\partial t} = 0,$$ (A25)

$$L_4 \frac{\partial u}{\partial t} = 0,$$ (A26)

$$L_5 \frac{\partial u}{\partial t} = 0,$$ (A27)

$$L_6 \frac{\partial u}{\partial t} = 0,$$ (A28)

$$L_7 \frac{\partial u}{\partial t} = 0,$$ (A29)

$$L_8 \frac{\partial u}{\partial t} = 0.$$ (A30)

The general approach in our simulation is that the characteristic equations along the outgoing characteristics are chosen (compatibility equations). The nonreflecting boundary conditions are chosen along the incoming characteristics. The nonreflecting boundary conditions and compatibility equations are combined into a complete system to determine the values of all the dependent variables on the boundary.

For the bottom boundary, we have the following situations.

1. When $v_n < c_f$, all projected characteristics are outgoing, equations (A15)-(A22) are used.
2. When $-c_f < v_n < -b_n$, there are seven outgoing and one incoming projected characteristics, equations (A15)-(A21) and (A30) are used.
3. When $-b_n < v_n < -c_f$, there are six outgoing and two incoming projected characteristics, equations (A15)-(A19) and (A21), (A28), and (A30) are used.
4. When $-c_f < v_n < 0$, there are five outgoing and three incoming projected characteristics, equations (A15)-(A17) and (A19), (A21), (A26), (A28), and (A30) are used.
5. When $0 < v_n < c_f$, there are three outgoing and five incoming projected characteristics, equations (A17)-(A19) and (A21), (A23), (A24), (A26), (A28), and (A30) are used.
6. When $c_f < v_n < b_n$, there are two outgoing and six incoming projected characteristics, equations (A19), (A21), (A23)-(A26), (A28), and (A30) are used.
7. When $b_n < v_n < c_f$, there are one outgoing and seven incoming projected characteristics, equations (A21), (A23)-(A28), and (A30) are used.
8. When $v_n = c_f$, all projected characteristics are incoming, equations (A23)-(A30) are used.
9. Top boundary treatment is similar to that in bottom boundary.

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